Constructive Ambiguity in Chapter 11[1]

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Abstract

The paper shows that ex ante institutional ambiguity in Chapter 11 bankruptcy protection can strengthen the protection to debtholders and firm welfare, and at the same time mitigate the moral hazard by distressed and bankrupt firms. We offer a dynamic model which describes a game between a distressed company and the bankruptcy court, taking into account that the court can implement a mixed strategy by randomly drawing judges that exercise different degree of power over the firm's operating decisions, as well as grant or do not grant the "automatic stay" extension of bankruptcy protection. "Optimal ambiguity" about the bankruptcy judge generate different effects in the favorable and unfavorable markets for the firm's assets. In bad markets, when the market offers low prices for assets that the firm can sell, Chapter 11 should optimally incorporate some degree of ambiguity which, if properly designed, can increase the firm value, ex ante, by alleviating incentives to forego asset sales and premature bankruptcy filing. If instead, there is an expectation that the bankruptcy judge always takes over firm's decisions to maximize firm welfare, rather than equity, then such certainty will incentivize shareholders to liquidate prematurely without selling assets, and not entering bankruptcy. On the other hand, if the judge does not have any power over operating decisions, such certainty leads to premature Chapter 11 bankruptcy. In good markets, the best outcome for the firm welfare is to sell part of its assets outside bankruptcy and to use sale funds to recapitalize. The ambiguity of bankruptcy courts can exacerbate moral hazard, where shareholders of the distressed firm may sub-optimally choose not to sell outside bankruptcy, but instead prematurely pursue bankruptcy. These predictions offer policy implications for the design of bankruptcy courts, which are qualitatively new to the related literature.

1 Introduction

Conventional wisdom suggests that institutional ambiguity hinders corporate decisions, implying that decisions are more efficient if such ambiguity is minimized. There is also a common view that the government institutions– when involved–should be expected to always take actions that enhance social objectives, and this expectation should be clearly communicated to all market participants. In response, firms should make decisions anticipating that the government transacts in the best interest the social welfare at all times.

We challenge these views using the framework of Chapter 11 bankruptcy. We show that insti-

tutional ambiguity of the Chapter 11 process can be value-enhancing. We explore the ambiguity that the distressed firm faces when it files for Chapter 11 bankruptcy protection and draws a bankruptcy judge. When there is no uncertainty that the judge will actively manage the bankruptcy to maximize the firm welfare, then, ex ante, such certainty might not necessarily lead to highest values. Instead, courts should make use of some "constructive ambiguity" credibly communicating uncertain standards regarding the role of bankruptcy judge. Specifically, we consider the ambiguity about the bankruptcy judge's objectives and her power over the firm's operating decisions that she can exercise during the automatic stay period of Chapter 11. We also consider the uncertainty of whether the judge will grant the firm an extension of "stay" after the initial "automatic stay" period expires.

We offer a dynamic model that incorporates these uncertainties and gauge their effects on the operating decisions of firms in distress, before and after pursing bankruptcy. First, we consider two extreme types of judges. The first type we call the "hands-on" (pro-active or powerful) judge who not only maintains the Absolute Priority Rule (APR) during bankruptcy and other restrictions that Chapter 11 imposes on the firm, but also exerts her power over the firm's operating decisions which she exercises with the objective of maximizing the total welfare of the bankrupt firm, i.e., total value of debt and equity. The judge's decisions include when and whether to force the bankrupt firm to sell some of its assets to outside markets and thus, recapitalize debt using sale funds, and whether and when to emerge from bankruptcy, or to liquidate. The other judge type is a "hands-off" type. Such judge does not affect the firm's operating decisions during bankruptcy, exerting no control over decisions to sell or not to sell assets, but only enforcing the APR during the protection period. During the automatic stay with such judge presiding, the firm itself chooses its operating decisions to maximize the equityholders' value, not the firm value.

We also, consider uncertainty of whether the judge grants or not the extension of automatic stay after the initial period (usually 120 days) expires, to a total duration of 18 months since initial filing. Before the distressed firm files for bankruptcy, the type of the judge is not known to the firm and the type is randomly drawn only after the firm enters bankruptcy and the judge is assigned. The probabilities of the two uncertainties are known to the markets and the firm, where the probability to receive the approval for stay extension is not correlated with the power of the judge.

In the model, the firm transacts in the best interest of shareholders, and the firm fully anticipates

these uncertainties of the Chapter 11 bankruptcy. The firm's earnings are stochastic and the firm continuously makes operating decisions depending on the firm's earnings and the market price for assets. The firm decides whether to continue operations with all assets, or to sell assets outside bankruptcy and recapitalize using sale funds. Alternatively, when earnings fall, the firm can file for Chapter 11 bankruptcy or fire-sale liquidate its assets. If the firm enters Chapter 11 and a hands-off judge is assigned, the firm fully controls the decisions on whether to continue operations within the protection period, or to sell the assets within bankruptcy and reduce its debt, or to emerge from bankruptcy without asset sales, or to liquidate. If the firm draws a hands-on judge, these decisions are under the full control of the judge. As such, the firm's shareholders always prefer a hands-off judge and the extension of the automatic stay.

The main result is that the ex ante ambiguity about the bankruptcy judge types in the Ch11 generate different effects in the favorable and unfavorable markets for the firm's assets. In bad markets, the market price for assets is relatively low, and the proceeds from sales would not be enough to reduce the firm's leverage ratio. In other words, during bad markets, the firm in the post asset-sale period will be even more leveraged than before sales. Therefore, the shareholders of the distressed firm are reluctant to sell assets, even if it will increase the overall firm value. The intuition is that sales proceeds cannot be channeled to shareholders but have to be used to reduce firm's debt, disproportionately benefiting debtholders at the expense of the shareholders. We call it "reversed underinvestment" problem. In contrast, in favorable market conditions, the asset sale would recapitalize the firm to a lower leverage ratio. However, the "reversed underinvestment" problem might still remain "in play", if the market price for assets is not sufficiently high, and the shareholders might still not opt to sell assets. Only if the price is high enough, assets sale can benefit both debt and equity, and the "reversed underinvestment" is gone.

In bad markets, the "no ambiguity" cases do not necessarily lead to the highest firm welfare. In fact, some uncertainty can be value enhancing. The key idea is that the decisions of whether and when sell assets outside bankruptcy, and whether and when to file for bankruptcy protection are fully under the control of the distressed firm that makes decisions to maximize the shareholder stake, not the total welfare of the firm. In bad markets, the bankrupt firm is likely not to sell its assets even though asset sales can increase the firm's overall value. The a value is preserved because the portion of assets sold at a market price, and not subject to fire-sale liquidation. But the sale at market prices can benefit debtholders at the expense of shareholders. If there is no uncertainty

that the judge is of "hands-on type", the distressed firm transacting in the best interest of the shareholders may sub-optimally choose to not to declare bankruptcy, forego the asset sales, and liquidate prematurely. Such premature liquidation without selling assets and not entering the bankruptcy incurs higher costs to the firm welfare. The reason why the firm is not incentivized to pursue Chapter 11 is that that filing incurs transaction and legal expenses that are incurred by shareholders, but under the "hand-on judge", the value channeled to shareholders is likely to be zero. These is because the "hands-on" judge will likely force the asset sales of division 2 and then immediately liquidate the remaining assets. Thus, expectation that the judge will alway act in the best interest of the firm (or social) welfare is not lead to best outcmes.

On the other hand, if the firm fully anticipates that the judge is the "hands-off" type without ambiguity, the firm is incentivized to file for Ch11 protection prematurely, i.e., when the firm is still not sufficiently distressed. Premature bankruptcy filing will hurt the firm value even more than a possible liquidation. These higher losses occur due to the shareholders' tendency to stay in bankruptcy without assets sales for the entire protection period, gradually cumulating distress costs and legal expenses which are function of time in bankruptcy, but still likely ending up in liquidation. Similarly, if the extension is always granted without ambiguity, the firm has incentives to enter bankruptcy even more prematurely. Premature bankruptcy filing reduce firm value due to firm likely staying in bankruptcy for a longer period. A longer stay duration increases optionality value for shareholders because there is always a chance that the firm recovers. However, financial distress costs accumulated in bankruptcy hurt debtholders disproportionately harsher if the firm ends up in fire-sale liquidation. But if the firm anticipates no extension with no ambiguity, it will not file and will liquidate without asset sales which could be a better outcome for the firm welfare.

The highest firm value is reached when there is an uncertainty about the judge types to be assigned, i.e., a mixed equilibrium. If the likelihood of drawing the hands-off judge is material, but not too high, and the probability of extension is high enough, the distressed firm will take chances and pursue bankruptcy hoping to draw a "hands-off" judge. If, instead, the hand-on judge is drawn, the judge will force the firm to sell assets, which will preserve some the value of the firm, but will wipe out the shareholders. The model offers a number of recommendations regarding optimal ambiguity design across the two dimensions. For example, if a court is populated by a larger fraction of "hands-on" judges, then the firm value reaches its maximum, if there is a higher chance for the extension of automatic stay. Conversely, if the court has larger fraction of hands-off

judges, to compensate, the probability of stay extension has be lower.

The model predictions are different when the market offers a good price for the firm's assets. In good markets, ambiguity about the bankruptcy judges may cause moral hazard and produce adverse effects. Specifically, in good markets, the firm value is maximized if the firm sells assets outside the bankruptcy. If the firm expects a hands-on judge with 100% probability, not pursuing bankruptcy is endogenously best decision for shareholders. However, if there is a material chance that hands-off judge can be assigned, combined with the expectation of a longer automatic stay, such combination will create moral hazard, which can reduce the firm value. The intuition is that such ambiguity may induce the shareholders of the distressed firm to forego the asset sales outside bankruptcy, but instead, to declare bankruptcy, even though the assets sale will maximize the total firm welfare. If hands-off is indeed assigned, costs associated with bankruptcy cumulate gradually during the bankruptcy, and the shareholders gradually become less and less incentivize to sell assets due to cumulated losses and unpaid interest, leading to a higher likelihood of liquidation without sales. Powers and Tsyplakov (2023) call such variant of moral hazard "a reverse" debt overhang or "reverse underinvestment" problem, where the equity holders are reluctant to sell part of the firm because funds collected from sales cannot be paid to shareholders, but have to be used to reduce firm's debt, disproportionately benefiting debtholders at the expense of the shareholders.

We should stress that we are not inventing the concept of courts ambiguity. The legal literature recognizes that legal and institutional ambiguities are essential elements of the US courts. Bankruptcy literature stresses that the US courts – as well as bankruptcy courts– are composed with judges that have different judicial philosophy over how legal process should be conducted. Our assumption about different types of bankruptcy judges is well-justified by the empirical literature. These studies show that "judge" fixed effect are statistically significant in the regressions for bankruptcy outcomes. For example, several papers (CITE) point out that some judges are more prone to liquidate the bankrupt firm during the bankruptcy protection, while others are more prone to reorganize it.

The ideas of ambiguity in bankruptcy courts is reminiscent to the concept of "Constructive Ambiguity" strategy in banking regulation. The main argument is that the bank regulator should not pronounce in advance which of large banks would be classified as too-big-to-fail and be always rescued. This "constructive ambiguity" often refers to the power that regulator has upon a failed bank to accept or reject the financial assistance and bank bailout, implying that such strategy can,

ex-ante, reduce the concern of moral hazard and risk shifting. (CITE XXXX) We should stress that there is a fundamental institutional difference between the concept of "Constructive Ambiguity" in the framework of Bank Supervisor and that in the framework of Chapter 11 bankruptcy. The mechanism of bailing out banks is structurally quite different from the mechanism of reorganizing bankrupt firms. First, banking regulators can often decide when to step in and rescue the bank or reject the bank's request for assistance. In Chapter 11, it is a distressed firm itself that decides when to declare bankruptcy, as involuntary bankruptcies are quite rare. Second, banks do not stop paying interest to depositors while being under the bail-out process. In contrast, the "automatic stay" allows the bankrupt firm to suspend interest payments and other liabilities for the duration of protection.

The remainder of the paper is organized as follows. The next section develops a theoretical model of the firm and the bankruptcy process. Section 3 reports model calibration. Section 4 presents numerical results for different combinations of the market price for assets and degree of uncertainties in Chapter 11, and the last section concludes the paper. Some stochastic control problems are formulated in Appendix A.

2 Model

2.1 Summary of the Model

We first present a continuous-time, infinite horizon model of firm that has two distinct divisions, and has no debt. Each division has its own production output and a cost structure. Both divisions continuously generate earnings calculated as revenue minus constant production costs, where revenue is stochastic. The firm pays out all its earnings as dividends after it pays taxes. The firm has an option to sell assets of division 2 to the outside buyer for a known market price. When the firm's revenue declines, it can become optimal for the firm to sell assets of Division 2. After the sale, the firm will continue to operate with Division 2. The sales proceeds will be paid to the shareholders. If revenue declines well below production costs, the firm can abandon the operations and close down the firm. Depending on the parameters, the firm can close either before or after the sales of division 2. We assume that the recovery value is zero when the firm closes down.

We then consider this firm being leveraged and operating with a perpetual coupon debt. The

periodic coupon payment is tax deductible.[2] When the firm's revenue declines its leverage increases and the firm can choose to sell assets of Division 2.[3] All funds from the sale have to be used to reduce the firm's debt and there are no payouts to the shareholders from sales proceeds unless debt is paid down in full. The debt is redeemed at the face value, i.e., at no discount is given to shareholders, implying that a strict version.[4] We assume that asset sales outside the bankruptcy can occur only if after debt restructuring, the firm emerges with non-zero equity as going concern. In other words, we disallow sales outside the bankruptcy that turns into an immediate liquidation, which by itself cannot be optimal for the shareholders.

If debt is not big enough, or market price for assets is high enough, by selling a fraction of the firm assets, the firm can reduce the firm leverage ratio by a larger fraction. After asset sales, the firm continues operations with the remaining assets of Division 1, and remaining debt. It can subsequently liquidate (Chapter 7), if revenue declines further. In case of liquidation, the equityholders receive zero and debtholders recover the firm's asset after incurring proportional deadweight costs applied to the remaining assets. This deadweight costs describe the fact that assets are often sold piece-meal at fire-sale prices.

The firm's shareholders may sub-optimally prefer to liquidate without asset sales and without entering the bankruptcy. The liquidation may be a preferred outcome if the sale price for assets is too small or debt is too big, so that the firm after asset sale would not reduce firm's leverage from what it had before. In such cases, optimal sales boundary is below the liquidation boundary. By assumption, at liquidation without selling division 2, the firm's going concern value is not preserved.

Alternatively, instead of liquidation, the firm can choose to file for bankruptcy. Bankruptcy filing incurs instant fixed transaction and administrative expenses. After filing for bankruptcy, the bankruptcy court grants the firm "an automatic of stay" for a some initial period (120 days). During automatic stay, the firm stops paying out interest payments (coupons) and cannot distribute dividends. The unpaid coupon payments are accumulated with interest on an account. The revenue net of production costs are also cumulated (with interest) on a separate account. The entire time

[2] Effectively, I assume that the firm cannot change its dividend policy and cannot retain any part of its earnings as cash to use for its future interest payments or investments. Allowing cash holdings to vary over time will significantly complicate the model analysis because it will increase the dimensionality of the problem and will create an additional decision for the firm.

[3] We do not differentiate whether the firm carves-out the division as a stand-alone company and then sell a newly-issued stock, or simply sells its assets directly to an outside buyer.

[4] We have also analyzed the version of the model that incorporates some violation of absolute priority rule and some debt forgiveness. The main results regarding assets by bankrupt companies still remain qualitatively the same.

the firm stays in bankruptcy protection, it incurs continuous costs associated with financial distress and administrative expenses until the firm emerges from bankruptcy. The firm can emerge from the bankruptcy before or at the end of the protection period. If the firm sells assets it immediately emerges from bankruptcy and continues operations with the remaining assets and lesser debt. If there is an excess funds left after all unpaid coupons are paid back, they are distributed to the shareholders. If there is not enough net earnings account to pay back all unpaid coupons, the shareholders can raise equity. If it is not optimal for the shareholders to raise equity, the firm liquidates and the net earnings account and the firm are transferred to debtholders after the liquidation costs are applied to the firm's assets (but not to the net earnings account). If the firm emerges from the bankruptcy as ongoing concern (with or without asset sales), the shareholders retain the option to liquidate the firm in a subsequent periods.[5]

The firm pursuing bankruptcy faces two types of uncertainties that depends on the type of bankruptcy judge and his/her judicial philosophy. i.e., the way in which a judge interprets the law. First, after an initial "automatic of stay" ($T_1 = 120$ days) is expired, the judge can subsequently extend it for another period (for up to $T_2 = 18$ months).[6] We assume two extreme cases of judge types: one judge always grants the firm an extension, regardless of the firm's conditions, while the other never gives an extension. The probability that the firm draws a judge who (does not) grants an extension is known to the firm and to the market before filing. However, whether the firm will draw such a judge is revealed only after filing. The duration of the initial automatic stay and the duration of extension are both known to the firm and to the market.

The second uncertainty is related to the control that the judge exerts over the firm's operating decisions within the automatic stay period. Specifically, the decision and the timing of asset sales and emergence from bankruptcy may not always be under a full control of the shareholders. Whether the firm has such a control depends on the type of bankruptcy judge that the firm draws after the filing. We consider two extreme types of bankruptcy judges: the "hands-off " judge and the "controlling" judge. The "hands-off" judge does not exert any power over firm's operating

[5] For technical simplicity, we assume that the firm after emerging from bankruptcy cannot enter bankrutcy again. This is a common assumption of the related literature. In other words, the firm has only a one-time option to seek bankruptcy protection. Also, if the firm emerges from the free-fall Chapter 11 without asset sales, it cannot subsequently sell assets.

[6] The US Bankruptcy Code gives the debtor the "automatic stay" period for the duration of 120 days from the bankruptcy filing date. During this period the bankrupt firm has to formulate a reorganization plan. The judge normally exends the "stay" period if there is a reasonable progress in the firm's operations. The extension duration can not be give the firm the "stay: period longer that the total of 18 months after the bankruptcy filing.

decisions. Specifically, hands-off judge do not control whether or when the bankrupt firm sells its assets during the bankruptcy protection period and when the firm emerges from bankruptcy. With the "hands-off " judge, the equityholders can optimally time the asset sales and the emergence from bankruptcy to maximize equityholders' value. Under the "controlling" judge, the timing of the asset sales and the timing of emergence from the bankruptcy is fully under the discretion of the judge, who maximizes the total value of the firm. The controlling judge can "order" asset sales any instant during protection period, even if the equityholders value declines to zero after the sales. In other words, the bankruptcy judge can "force" the bankrupt firm to sell Division 2 and then immediately force liquidation. In that case, the remaining Division is liquidated and equityholders are wiped out. It is important to clarify that the controlling judge does not necessarily favor debt over equity per se during the bankruptcy, but rather protects the total welfare of the firm. Finally, for both judge types, the APR is fully complied. The probability of selecting the "hands-off" judge or a controlling judge is known to the market. The assignment of the bankruptcy judge type is random and the type revealed only after the firm filed for Ch11, and it is independent from whether the judge grants an extension.[7]

2.2 The Firm's Revenue and Asset Sales

The firm operates two divisions of the firm that produce combined continuous revenue stream, p, described by the following stochastic process:

$$\frac{\mathrm{d}p}{p} = (r - \alpha)\mathrm{d}t + \sigma_p \mathrm{d}W_p, \tag{1}$$

where W_p is a Wiener process under the risk neutral measure Q, σ_p is the instantaneous volatility coefficient, r is the risk free rate, which is assumed to be constant, and α ($\alpha > 0$) is the convenience yield.

The divisions produce fractions of revenue Q_1 and Q_2, respectively, where $Q_1 + Q_2 = 1$. Each division has its own continuous production costs of c_1 and c_2, respectively. So the total production

[7] For technical simplicity, we assume that the firm does not have an option to enter Chapter 11 multiple times. This is a common assumption of the related literature. In other words, the firm has only a one-time option to seek Chapter 11 bankruptcy and cannot enter Chapter 11 again after in emerges from bankruptcy earlier. Also, if the firm emerges from Chapter 11 without asset sales it cannot subsequently sell assets under prenegotiated deals or through Section 363 in subsequent periods.

cost is $c = c_1 + c_2$, and the firm's net earnings before interest and taxes are equal $(p \cdot Q_1 - c_1)$ and $(p \cdot Q_2 - c_2)$, respectively, per division 1 and 2.

2.3 Debt, Corporate Taxes and Dividends

We assume that the firm issues perpetual coupon debt with a periodic coupon payment d.[8] Firm's net earnings are taxed continuously at a constant corporate rate, λ, where periodic debt coupon payments d are tax deductible. The firm's instantaneous after tax dividends are:

$$(1 - \lambda) \cdot [p \cdot Q' - c' - d'], \tag{2}$$

where d' represents periodic debt payments, and output Q' equals either $Q_1 + Q_2 = 1$, before the asset sales, or Q_1 after the sales. Similarly, the variable $c' = c_1 + c_2$, or c_1.

2.4 Unlevered Firm

We denote S as a sale price for assets of division 2. We first start with the valuation of the unlevered firm after the asset of division 2 are already sold. The firm's after tax earning are $(1-\lambda)\cdot[p \cdot Q_1 - c_1]$. If revenue declines to critical value p_A, the firm abandons the assets and closes down with zero recovery value. The price at which the firm operating with division 1 only abandons operations is

$$p_{A_q} = \frac{c_1}{Q_1}\left(\frac{-\beta_2}{(1-\beta_2)}\right)\frac{\alpha}{r}, \tag{3}$$

and the unlevered firm value is after asset sales is:

$$V_{Uq}(p) = (1 - \lambda) \cdot [(Q_1 \cdot p/\alpha - c_1/r) - (Q_1 \cdot p_{A_q}/\alpha - c_1/r) \cdot (p/p_{A_q})^{\beta_2}],\ p > p_{A_q}, \tag{4}$$

$$\text{where } \beta_2 = 0.5 - (r-\alpha)/\sigma^2 - \sqrt{[(r-\alpha)/\sigma^2 - 0.5]^2 + (2r/\sigma^2)} < 0. \tag{5}$$

[8] I effectively assume that the firm is not allowed to sell zero coupon bonds. Lack of zero coupon corporate bonds in practice justifies this assumption.

With no debt, the sale proceeds are paid out to the shareholders of the all-equity firm. Thus, the value of the unlevered firm before it sells assets of division 2 for a sale price of S has to satisfy the following equation:

$$V_U(p) = [(1-\lambda)\cdot(p/\alpha-(c_1+c_2)/r)-\{(1-\lambda)\cdot(p_{us}/\alpha-(c_1+c_2)/r)-V_{Uq}(p_{us})-S\}\cdot(p/p_{us})^{\beta_2}],\ p > p_{us} > p_{A_q}, \tag{6}$$

where revenue p_{us} which triggers asset sales of division 2 has to satisfy the following equation:

$$\frac{\partial V_U(p_{us})}{\partial p} = \frac{\partial V_{Uq}(p_{us})}{\partial p},\ \text{and}\ V_U(p_{us}) = V_{Uq}(p_{us}) + S, \tag{7}$$

which can be solved numerically.

2.5 Levered Firm that can sell assets outside bankruptcy or liquidate

We now consider a levered firm that operates with debt which has face value F and a continuous interest payment d, which are tax-deductable. We first consider case, where the firm can instantly sell assets or liquidate without asset sales, but cannot file for bankruptcy. The shareholders optimally time the asset sale at the critical revenue at which maximizes the firm's equity. When the firm sells division 2, all the sales proceeds S have to be used to reduce the firm's debt. We assume that the size of the debt is large enough so that the sale price, i.e., $S < F$. We assume that the firm has to repay the debt at a risk free rate so that face value declines from F to F_1, where $F_1 = F - S$. After the asset sale, the firm's coupon declined from d to $d_1 = d - S \cdot r$.[9]

Similar to the previous section, we start the analysis of the levered firm after it already sold assets of division 2 operates with Division 1. After asset sales, the firm's continuous interest payment is d_1. The after tax residual dividends are $(1-\lambda)\cdot[p\cdot Q_1 - c_1 - d_1]$. If revenue declines to critical value p_{d_1}, at which the equity value is zero, the shareholders optimally liquidate the remaining division, and the debtholders take control of the firm after incurring proportional default costs, DC. At liquidation, the shareholders have zero recovery value. It is easy to show that the price at which the shareholders optimally default is

[9] We parametrize the model so that the initial debt size is larger than proceeds from assets sales, i.e., $\frac{d}{r} > S$, so after the asset sales, the firm continues with a non-zero debt.

$$p_{d_1} = \frac{c_1 + d_1}{Q_1}(\frac{-\beta_2}{(1-\beta_2)})\frac{\alpha}{r}. \tag{8}$$

The values of Debt and Equity after the sale occurs satisfy the following equations:

$$E_1(p, d_1) = (1-\lambda) \cdot [Q_1 \cdot p/\alpha - c_1/r - d_1/r) - (Q_1 \cdot p_{d1}/\alpha - c_1/r - d_1/r) \cdot (p/p_{d1})^{\beta_2}], \tag{9}$$

$$D_1(p, d_1) = d_1/r - [d_1/r - (1-DC) \cdot V_{Uq}(p_{d_1})] \cdot (p/p_{d_1})^{\beta_2}, p > p_{d_1} > p_{A_q}. \tag{10}$$

Before the asset sale, the equity and debt value has to satisfy the following equation:

$$E_0(p, d) = (1-\lambda) \cdot (p/\alpha - (c_1 + c_2 + d)/r) - [(1-\lambda) \cdot (p_s/\alpha - (c_1 + c_2 + d_1))/r - E_1(p_s, d_1)] \cdot (p/p_s)^{\beta_2}, \tag{11}$$

$$D_0(p, d) = d/r - [d_1/r - D_1(p_s, d_1)] \cdot (p/p_s)^{\beta_2}, p > p_s > p_{d_1}, \tag{12}$$

where p_s is revenue at which the equity holders optimally sell assets of division 2, so that the equity value is maximized. Thus the following smooth pasting and maximization conditions:

$$\frac{\partial E_0(p_s, d)}{\partial p} = \frac{\partial E_1(p_s, d_1)}{\partial p}, \text{and } E_0(p_s, d) = E_1(p_s, d_1), \tag{13}$$

which can be solved numerically for p_s. At the sales trigger p_s, the debt value is $D_0(p_s, d) = D_1(p_s, d_1) + S$.

If the sale price S is small relative to debt size, the value of the equity can become zero or negative at p_s, which satisfies the condition 13. In such case, the equity value is maximized if the firm forego the option to sell assets and liquidates when revenue declines further to $p_{ns}, (p_{ns} > p_s)$, at which the equity value becomes zero, where subscript ns stands for the liquidation trigger with "no sale" of assets. In other words, for low enough sale price S (or equivalently a large enough debt), the sales option is worthless for the shareholders because the liquidation decision (without asset sales) makes the equity value, $E_0^{ns}(p, d)$, higher then for the case with asset sales,

i.e., $E_0(p,d) < E_0^{ns}(p,d)$, for any $p > p_{ns}$,where:

$$E_0^{ns}(p) = (1-\lambda) \cdot [(p/\alpha - c/r - d/r) - (p_{ns}/\alpha - c/r - d/r) \cdot (p/p_{ns})^{\beta_2}], p_{ns} > p_{A_q}, \tag{14}$$

$$p_{ns} > p_s, p_{ns} > p_A, \tag{15}$$

p_{ns} is the optimal liquidation trigger at which the equityholders liquidate both division of the firm with no asset sales. It is easy to show that the optimal liquidation trigger is

$$p_{ns} = (c+d)(\frac{-\beta_2}{(1-\beta_2)})\frac{\alpha}{r}. \tag{16}$$

The debt value for the case when $p_{ns} > p_s$ is

$$D_0^{ns}(p,d) = d/r - [d/r - (1-DC) \cdot V_U(p_{d_{ns}})] \cdot (p/p_{ns})^{\beta_2}]. \tag{17}$$

Thus, the debt and equity value of the firm's that can instantly sell assets, but has no option to pursue bankruptcy is

$$E_{No\_Filing}(p,d) = \max \begin{cases} E_0(p,d), \text{ if } p > p_s > p_{d_1},\ p_s > \ p_{ns}, \text{the firm sells assets} \\ E_0^{ns}(p), \text{ if } p > p_{ns} > p_s, \text{ i.e.,, the firm is} \\ \text{liquidated without asset sales.} \end{cases} \tag{18}$$

$$D_{No\_Filing}(p,d) = \begin{cases} D_0(p,d), \text{ if } p > p_s > p_{d_1},\ p_s > \ p_{ns}, \text{the firm sells assets} \\ D_0^{ns}(p), \text{ if } p > p_{ns} > p_s, \text{ i.e., the firm is} \\ \text{liquidated without asset sales.} \end{cases} \tag{19}$$

Proposition: For a critical level of asset sale price $S^* = \frac{Q_1 d - Q_1(c_1+c_2)+c_1}{r}$, for which $p_{d1} = p_{ns}$, and $E_0(p_{d1},d) = E_0^{ns}(p_{d1},d) = 0$, the shareholders are indifferent between liquidating the firm without selling assets or selling assets and then immediately liquidating the firm. If $S < S^*$, then

$p_{d1} > p_{ns}$, and the shareholders optimally choose not to sell assets, but liquidate both divisions as soon as revenue declines further to p_{ns}.

Proof of Proposition: if the equity value of the option to sell assets is higher than the equity value with the option to liquidate, then, $([-(p_s/\alpha - (c+d))/r + E_1(p_s, d_1)] \cdot (1/p_s)^{\beta_2} > [-(p_{ns}/\alpha - c/r - d/r) \cdot (1/p_{ns})^{\beta_2}]$. If $p_s > p_{ns}$, then, $E_1(p_s, d_1) > 0$, which implies $p_{d1} < p_s$. Hence, $p_{ns} < p_{d1}$. Otherwise, the above equation cannot hold. A different way to prove is the following. If $p_{ns} > p_{d1}$, then at p_{ns}

$E_1(p_{ns}, d_1) > E_0^{ns}(p_{ns}) = (1-\lambda) \cdot [(p_{ns}/\alpha - c/r - d/r) - (p_{ns}/\alpha - c/r - d/r) \cdot (p_{ns}/p_{ns})^{\beta_2}] = 0.$ Then,

$(1-\lambda) \cdot [Q_1 \cdot p_{ns}/\alpha - c_1/r - d_1/r) - (Q_1 \cdot p_{d1}/\alpha - c_1/r - d_1/r) \cdot (p_{ns}/p_{d1})^{\beta_2}] > 0,$

and hence at p_{ns} the following has to hold:

$[Q_1 \cdot p_{ns}/\alpha - c_1/r - d_1/r) - (Q_1 \cdot p_{d1}/\alpha - c_1/r - d_1/r) \cdot (p_{ns}/p_{d1})^{\beta_2}] > 0$, which implies that $p_{ns} > p_{d1}$. At the debt payment size for which $p_{ns} = p_{d1}$, the shareholders is indifferent between liquidating the firm or executing the asset sales.

From these equations, the critical sale price, S^*, has to satisfy $c_2 - c_1 = d^*(1-Q_1) - S \cdot r$, $(c_2 - c_1 + S \cdot r) = d^*(1-Q_1)$, and therefore $S^* = \frac{Q_1 d - Q_1(c_1+c_2)+c_1}{r}$.[10]

Thus, if the sale price exceeds S^*, the option to sell assets leads to a higher equity value than the option to liquidate the firm. Otherwise, if the debt size $S \leq S^*$, the shareholders liquidate the firm without selling assets.

2.6 Firm Files for Chapter 11 bankruptcy and Draws Hands-off Judge

We first start the analysis for the case where the protection period T (T_1 or T_2) is known to the firm and the markets and the judge is hand-off type. The timing of the Chapter 11 filing is a choice of the shareholders. The automatic stay is enacted in bankruptcy, and the firm does not make interest payments and do not pay dividends for the duration of the protection span or until the firm is liquidated or emerges out of bankruptcy. We assume that Chapter 11 is associated with

[10] If the chapter 11 is not possible, the other way to preempt liquidation is to renegotiate that the firm sells assets at the product price above p_s but let it pay out some of the sale proceeds to equityholders. This would be a forbearance (a violation) of the debt covenant. If such forbearance is optimally designed, it can potentially enhance the firm value by incentivizing the firm to sell assets earlier before entering a deep distress stage. Another alternative to incentivize the firm to sell assets is to have debtholders accept a partial write-down of debt. The model can handle these extensions too, but the renegotiation of debt covenants or debt forgiveness is outside the main focus of this paper.

various costs for the bankrupt firm. First, at the time of filing, the firm pays a one-time transaction cost which corresponds to an one-time legal, advisory and filing fees. Second, for the length of the protection period the bankrupt firm incurs continuous costs that are fraction of the firm's earnings, $p \times c_d$, where c_d is positive constant. These costs are normally associated distress costs due to a possible losses of employees and customers as well as ongoing administrative and legal expenses, both of which can be high because of the uncertainty of bankruptcy outcome.

The unpaid interest payments as well as net earnings are cumulated on separate accounts and grow with risk free rate. The firm can emerge from bankruptcy in two ways: First, any time before protection period expires, the firm can decide to sell assets. When the firm sells assets, the proceeds of sales are paid to reduce the size of debt, and the account for net earnings have to be used to payoff all unpaid interest payments cumulated on the account, with any residual paid out to shareholders. If after the asset sales, there is not enough funds on the net earning account to pay off all unpaid coupon, the remaining unpaid coupons have to be paid by newly issued equity.

Second, the firm can decide to emerge from bankruptcy without selling assets any time at or before the protection period expiries. For most cases, the firm that optimally decides to emerge without asset sales will become less leveraged in the post-bankruptcy protection period. This because such decision is optimal only if revenue recovers significantly during the bankruptcy protection period. As mentioned above, after the firm exits the bankruptcy without asset sales, it will have no further option to declare bankruptcy again or to sell assets.

Thus, the net earning account N, is described as

$$\mathrm{d}N(t) = (rN + p - p \times c_d - c_1 - c_2 - (\lambda) \cdot [p - c_1 - c_2 - d])\mathrm{d}t, \text{ and where } N(0) = 0. \tag{20}$$

The last term represents taxes (or tax credit) that the firm cumulates during the protection period. The dynamics for the account for unpaid interest payments during the protection period, $U(t)$, is described as:

$$dU(t) = (rU + d)\mathrm{d}t, \text{ and } U(0) = 0. \tag{21}$$

Its value can be calculated forward for each t, $t \leq T$, using a future value of continuously com-

pounded annuity: $U(t) = d\frac{e^{r\cdot t}-1}{e^r-1}$.

2.6.1 Firm Valuation Within the Protection Period

We first present the valuation for debt and equity inside the protection period given that the firm already filed for Chapter 11. For the size of interest payments d, the value of the firm's equity $E^T_{off}(p, N, U, t)$ within the protection period is a function of the revenue, p, net earning account, N; the account for unpaid interest U; and the time elapsed t since filing, $0 \leq t \leq T$, where T is the protection period duration, which can be either $T_1 = 0.333$ or $T_2 = 1.5$, if the extension is granted.

At the end of protection period, at $t = T$, the equity and debt values are known. The firm that emerges from bankruptcy with (or without) selling assets will have no more option to file for Chapter 11 in subsequent periods, but has an option to liquidate.[11] At $t = T$, the firm has an option to sell assets, or emerge from bankruptcy without selling assets, or liquidate, and the firm chooses the course of action that maximize its equity value. Specifically, the value of the firms's equity is equal the following:

$$E^T_{off}(p, N, U, T) = \max \begin{cases} E_1(p, d_1) + N(T) - U(T), & \text{the firm sells assets at } t = T. \\ E^{ns}_0(p) + N(T) - U(T), & \text{the firm emerges from bankruptcy without asset sales.} \\ 0, & \text{the firm is liquidated at } t = T. \end{cases} \tag{22}$$

The first term on the right-hand side describes the equity value at the end of the protection period when the firm chooses to sell assets at $t = T$. As assumed, the firm uses all sales proceeds to reduce debt. When the cumulated net earnings can not pay for all unpaid coupons, i.e., if the net earnings account is less than unpaid coupon account, i.e., $U(T) > N(T)$, the firm has to issue equity to cover the difference. The choice between emerging with or without sales of the assets is the one that maximizes the market value of the firms's equity. The debt value at $t = T$ depends on the firm's decisions:

[11] If the firm emerges without asset sales, there is no subsequent option to sell assets.

$$D^T_{off}(p,N,U,T) = \begin{cases} D_1(p,d_1) + S + U(T), \text{ the firm sells assets} \\ D^{ns}_0(p) + U(T), \text{ the firm emerges from bankruptcy without asset sales.} \\ (1-DC)V_U(p) + N(T), \text{ if } E_2(p,N,U,T) = 0, \text{ the firm} \\ \text{is liquidated at } t = T. \end{cases} \tag{23}$$

For time $0 \leq t < T$, the firm chooses its timing when to sell assets and when to emerge from Chapter 11 with the objective to maximize the market value of its equity, $E(p,N,U,t)$. The solution involves determining free boundary conditions that divide the state space (p,N,U,t) into three regions that characterize the firms's choices: the *no action region*, the *asset sales region*, and the *default region*.[12]

Using standard arbitrage arguments outlined in Merton (1974)[13], for $0 \leq t < T$, the value of the equity and debt in the *no action region* is given by the solution to the following PDE:

$$\begin{aligned} \frac{\sigma^2 p^2}{2}\frac{\partial^2 E_{off}}{\partial p^2} + (rN + p - c_1 - c_2 - (\lambda) \cdot [p - p \times c_d - c_1 - c_2 - d])\frac{\partial E_{off}}{\partial N} \\ + (rU + d)\frac{\partial E_{off}}{\partial U} - \frac{\partial E_{off}}{\partial t} - rE_{off} = 0, \\ \frac{\sigma^2 p^2}{2}\frac{\partial^2 D_{off}}{\partial p^2} + (rN + p - c_1 - c_2 - (\lambda) \cdot [p - p \times c_d - c_1 - c_2 - d])\frac{\partial D_{off}}{\partial N} \\ + (rU + d)\frac{\partial D_{off}}{\partial U} - \frac{\partial D_{off}}{\partial t} - rD_{off} = 0, \end{aligned} \tag{24}$$

The equation is not time-homogeneous because the protection period has finite length, and the term $\frac{\partial E}{\partial t}$ represents the change in equity value due to continuous changes in elapsed time within the protection period. Thus, there is no steady state solution for the firm's values and no closed-form solution.

The shareholders optimally decide to sell assets if the net benefit of sales and subsequent emergence from the bankruptcy exceeds the equity value when the firm continues to operate under

[12] For brevity, we omit the discussion of the technical detail of boundary and "high contact" conditions that applied to the value function E. For details see Oksendal and Sulem (2007).

[13] Merton (1974) shows that in a complete market, any security, that is a function of the underlying security p and time t, can be replicated by continuously rebalancing a portfolio of futures contracts and risk-free bonds. Therefore the security value $V(p,t)$ has to satisfy the following equation $\frac{1}{2}\sigma^2 p^2 V_{pp} + (r-\alpha)pV_p - rV + CF(p,t) + V_t = 0$, where $CF(p,t)$ is cash flow rate to the security holders.

Chapter 11 protection. Similarly, the firm decides to emerge from bankruptcy without selling assets if the net benefit exceeds the continuation option. The equity values in this region can be determined by the following:

$$E_{off}(p,N,U,t) = \max \begin{cases} e^{-rdt}\mathbb{E}^Q\{E_{off}(p,N,U,t+dt)\}, \text{ firms continues to operate} \\ \text{under the Chapter 11 protection} \\ E_1(p,d_1) + N(t) - U(t), \text{ the firm sells assets and} \\ \text{emerges from bankruptcy} \\ E_0(p,d) + N(t) - U(t), \text{the firm emerges} \\ \text{from bankruptcy without asset sales.} \end{cases} \tag{25}$$

$$D_{off}(p,N,U,t) = \begin{cases} e^{-rdt}\mathbb{E}^Q\{D_{off}(p,N,U,t+dt)\}, \text{ firms continues to operate} \\ \text{under the Chapter 11 protection} \\ D_1(p,d_1) + S + U(t), \text{ the firm sells assets} \\ \text{and emerges from bankruptcy} \\ D_0^{ns}(p) + U(t), \text{the firm emerges from} \\ \text{bankruptcy without asset sales.} \end{cases} \tag{26}$$

where $\mathbb{E}^Q$ is the expectation operator under the risk neutral measure Q.

2.7 Firm draws a hands-on judge

In this section, we analyze the firm that draws a hands-on judge without uncertainty. The time line of this bankruptcy method is similar to the case with hands-off judge, with one important exception: when the firm enters the bankruptcy, all operating decisions are controlled by the judge. The evaluation for debt and equity is similar, but needs to be slightly adjusted, as the judge forces the firm to choose decisions to maximize the firm total value. The value of debt and equity (D_{on} and E_{on}) are calculated by incorporating the judge's decisions. At the end of the protection period, $t = T$, the value of the firm V_{on} under the hands-on judge is equal the following:

$$V_{on}(p,N,U,T)=\max\begin{cases}E_1(p,d_1)+N(T)+D_1(p,d_1)+S,\ \text{if } E_1(p,d_1)>0,\ \text{the judge}\\ \text{orders asset sale, the firm emerges from bankruptcy as going concern}\\ S+(1-DC)V_{Uq}(p)+N(T),\ \text{if } E_1(p,d_1)=0,\text{the judge}\\ \text{orders asset sale, and liquidates remaining assets}\\ E_0(p,d)+N(T)+D_0^{ns}(p),\ \text{the firm emerges}\\ \text{from bankruptcy without asset sales}\end{cases} \tag{27}$$

$$D_{on}(p,N,U,T)=\begin{cases}D_1(p,d_1)+S,\ \text{the firm sells assets}\\ \text{and emerges as going concern}\\ S+(1-DC)V_{Uq}(p)+N(T),\ \text{if } E(p,N,U,T)=0,\ \text{and the firm sells}\\ \text{assets and liquidated}\\ D_0^{ns}(p)+U(t),\text{the firm emerges from bankruptcy}\\ \text{without asset sales}\end{cases} \tag{28}$$

$$E_{on}(p,N,U,T)=\max\begin{cases}E_1(p,d_1)+N(T)-U(T), & \text{the firm sells assets}\\ & \text{and emerges as going concern}\\ E_0^{ns}(p)+N(T)-U(T), & \text{the firm emerges from}\\ & \text{bankruptcy without asset sales.}\\ 0, & \text{the firm is liquidated at } t=T.\end{cases} \tag{29}$$

For any time t, $0\le t<T$, similar to the case with hands-off judge, we solve for free boundary conditions that reflect judges's sale/no sale decision, the decision to emerge from bankruptcy or to liquidate. In the *no action region*, the value of the equity is given by the same PDE as in 24.The judge calls for asset sale if the firm value after emergence from the bankruptcy exceeds the its continuation value under protection period. The equity values in these regions can be determined

by the following:

$$V_{on}(p,N,U,t)=\max\begin{cases} e^{-rdt}\mathbb{E}^{Q}\{E_{on}(p,N,U,t+dt)+D_{on}(p,N,U,t+dt)\},\ \text{the firm} \\ \text{continues to operate under the protection period} \\ E_1(p,d_1)+N(t)+D_1(p,d_1)+S,\ \text{the firm} \\ \text{sells assets and emerges with less debt and smaller capacity.} \\ S+(1-DC)V_{Uq}(p)+N(T),\ \text{if } E_1(p,d_1)=0,\text{the judge} \\ \text{orders asset sale, and liquidates remaining assets} \end{cases} \tag{30}$$

2.7.1 Firm valuation before bankruptcy filing when the duration of automatic stay Chapter 11 and the judge type are random

We now assume that there is probability f with which the judge extends an "automatic of stay" from the initial 120 days ($T_1 = 0.33$ years) to the total 18 months ($T_2 = 1.5$ years). Also, the probability that the judge will be of a hands-on type is g.

Let's denote the revenue p_{Ch11} as a trigger at which the equity holders elect to file for the Chapter 11, taking into account probabilities of f and g. By assumption, when the firm files for Chapter 11, it pays one-time filing and administrative fees, C_{Ch11}. The values of debt and equity satisfy the following equations:

$$\begin{aligned} &E_{Ch11}(p,d)=(1-\lambda)\cdot(p/\alpha-(c_1+c_2+d)/r)- \\ &[(1-\lambda)\cdot(p_{Ch11}/\alpha-(c_1+c_2+d_1)/r)+C_{Ch11}-E(p_{Ch11},0,0,0)]\cdot(p/p_{Ch11})^{\beta_2},\ p>p_{Ch11}. \end{aligned} \tag{31}$$

where, $E(p_{Ch11},0,0,0)=(1-g)\cdot[(1-f)\cdot E^{T_1}_{off}(p_{Ch11},0,0,0)+f\cdot E^{T_2}_{off}(p_{Ch11},0,0,0)]+g\cdot[(1-f)\cdot E^{T_1}_{on}(p_{Ch11},0,0,0)+f\cdot E^{T_2}_{on}(p_{Ch11},0,0,0)]$, which is probability-weighted expected equity value. At the revenue p_{Ch11}, the value of the equity has to satisfy the smooth pasting and value maximization conditions:

$$\frac{\partial E_{Ch11}(p_{Ch11},d)}{\partial p}=\frac{\partial E(p_{Ch11},0,0,0)}{\partial p},\ \text{and}\ E_{Ch11}(p_{Ch11},d)=E(p_{Ch11},0,0,0)$$

which we solve numerically for p_{Ch11}. The debt value is

$$D_{Ch11}(p,d) = d/r - [d/r - D(p_{Ch11},0,0,0)] \cdot (p/p_{Ch11})^{\beta_2}, \text{ where}$$

$D(p_{Ch11},0,0,0) = (1-g)\cdot[(1-f)\cdot D^{T_1}_{off}(p_{Ch11},0,0,0) + f\cdot D^{T_2}_{off}(p_{Ch11},0,0,0)] + g\cdot[(1-f)\cdot D^{T_1}_{on}(p_{Ch11},0,0,0) + f\cdot D^{T_2}_{on}(p_{Ch11},0,0,0)]$ is a probability-weighted debt value. At $p = p_{Ch11}$, the debt value $D_{Ch11}(p_{Ch11},d) = D(p_{Ch11},0,0,0)$.

2.7.2 Valuation when the firm has the option to liquidate without filing, sell assets outside the bankruptcy, or file for bankruptcy

The value of the firm is calculated given that the equityholders will choose the action between 1) selling assets outside the bankruptcy, 2) liquidating without sale, 3) filing for bankruptcy, that maximize the value of equity:

$$E(p,d) = \max \begin{cases} E_{No\_Filing}(p,d), \text{ no filing is optimal, } p > p_{ns}, p > p_s, p > p_{Ch11}, \\ E_{Ch11}(p,d), \text{ filing is optimal,} \end{cases} \tag{32}$$

and, thus, depending on the equityholders' decision, the debt value is

$$D(p,d) = \begin{cases} D_{No\_Filing}(p,d), \text{ if } E_{No\_Filing}(p,d) > E_{Ch11}(p,d) \\ D_{Ch11}(p,d), \text{ otherwise.} \end{cases} \tag{33}$$

3 Model Results

3.1 Base Case Parameters

Most parameters values that describe the firm's cash earnings, interest rates, taxes have been already calibrated in numerous Leland-type models. To save the space we do not repeat calibration analysis and instead use parameters from the published papers. The volatility of revenue σ is set at 20%. The yield α is set at 7%. The risk free interest rate is set to $r = 7\%$, this means that the risk neutral drift of revenues is zero. We assume that the production capacity of Division 1 and Division

2 are the same, so that $Q_1 = Q_2 = 0.5$. The production costs are the same as well: $c_1 = c_2 = 1.5$, and $c = 3.0$. This implies that the two divisions have the same productivity. The sale price for the assets of division 2 is considered for $S = 11.0$ and $S = 12.0$. For the base case, we set the debt size with the (risk-free) face value at $D = 23.0$, and a coupon size $d = 1.61$, so that $D = \frac{d}{r} = 23$. For this parametrization, if the firm sells half of its assets for the price of $S = 11$, it can reduce the debt size by less than a half: from $D = 23$ to $D_1 = 23.0 - 11.0 = 12.0$. If the sale price $S = 12$, it can reduce the debt size by more than a half: from $D = 23$ to $D_1 = 23.0 - 12.0 = 11.0$.[14] For $S = 11$ and $S = 12$, the asset sale boundary p_s below (above) the liquidation boundary p_{ns}, but relatively close, so that the model produces non-trivial results.

There are several parameters, which have not been calibrated in the literature. These parameters describe transaction costs and legal expenses and transaction costs associated with pursuing bankruptcy. To calibrate these parameters, we collect data of total legal fees for the Chapter 11 bankruptcy. For the period XXX-XXX, the average fees measured as percentage of the firms assets are XXX. In the calculation of this ratio, we use the latest reported asset size of the firm before it filed for bankruptcy. In the base case, the firm sells half of its production capacity (division 2) for either $S = 11$, or $S = 12$, which implies that sale price of the firm can be approximated at $2.0 \cdot S$. Thus, for the base case, we assume that filing fees for Chapter 11 is $C_{Ch11} = (2.0 \cdot S) \cdot 1.0\%$. Proportional distress and administrative costs that the firm incurs during the bankruptcy period is $c_d = 10\%$ of the revenue p. This parametrization reflects the fact that "the value erodes" during the Chapter 11 bankruptcy duration.

As we mentioned, we assume in the model that the length of the protection period, T, is either $T_1 = 0.33$ year or if it is extended, $T_2 = 1.5$ years, which is randomly depends on the judge draw. The model allows us to focus on the how Chapter 11 uncertainty can affect decisions of the distressed and a bankrupt firm.

3.2 Model Results

3.2.1 Unlevered Firm

We first consider the properties of the solution of the unlevered firm in the base case. The calculations show that the firm optimally triggers the assets sales at revenue $p_{us} = 3.75$. After the

[14] As we assumed, the firm has to repurchase the debt at the risk-free face value of debt.

assets are sold, the firm continues operations with the remaining Division 1 and optimally closes its operations when revenue declines further to $p_{A_q} = 2.01$.

3.2.2 Base Case: Levered Firm with no Option to File for Bankruptcy

Second, we analyze a levered firm in the case where it has an option to sell assets outside bankruptcy or liquidate, and no option to pursue bankruptcy. For the base case parameters, the critical sale price is $S^* = \frac{Q_1 d - Q_1(c_1+c_2)+c_1}{r} = 11.5$, as analytically described in Proposition. We consider two cases, with sale prices below and above the critical level: $S = 11.0 < S^*$, and $S = 12.0 > S^*$. If $S = 11.0$, the firm's equityholders optimally choose to liquidate without selling assets. Otherwise, for sale price $S = 12.0 > S^*$, the firm's shareholders optimally choose to sell assets and then they might liquidate the remaining assets if the revenue declines further to p_{d_1}.[15]

From solutions of the optimization problem in 13, if $S = 11.0$, the revenue which triggers liquidation without asset sales is $p_{ns} = 3.095$. If $S = 12$, the assets sales boundary is $p_S = 3.14$.

3.2.3 The optimal choice between selling assets or liquidating outside the bankruptcy or filing for Chapter 11 bankruptcy

We evaluate the firm's at earning $p = 6.0$ which are significantly higher than earnings at which the levered firm triggers an asset sale and at which it liquidates.[16] The evaluation is done by assuming that, if earnings fall significant enough, the firm makes the optimal choice between selling assets outside the bankruptcy court, liquidating without sales, or filing for Chapter 11 bankruptcy protection. The decision at what stage of distress the firm files for bankruptcy depends on uncertainty of Chapter 11 and the sale price for the firm's assets.

The Case with Low Price for the Firm's Assets We first consider the case where the sale price $S = 11.0$ which is below the critical level $S^* = 11.5$, so that the liquidation boundary is above the sale boundary, meaning that the firm's shareholders (that has no option to file for Ch11) will

[15] Given that the sale price is $S = 12$, it means that the firm sells half of its production capacity ($Q_2 = 0.5$), and the sale funds can reduce the debt size (the coupon size) by at least half, i.e., if the face value (coupon size) declines from D to $D_1 = D - 12.0 < 11.5$, (from d to $d_1 < \frac{1.61}{2}$). This holds for the base case, where the debt (coupon size) declines by more then half. For the base case, the sale proceeds used to repay debt reduce the debt size from $D = 23.0$ to $D_1 = 11.0$.

[16] We are dealing with time-homogenouus equations. As such, it does not matter at which earning level we compare firm value under different cases as far as the earnings are higher than the liquidation, the sale boundary, and the filing for bankruptcy boundary.

choose to liquidate the firm without selling assets as soon as its earnings decline to $p_{ns} = 3.095$. The main question is whether the option to enter Chapter 11 can induce incentives to the firm to file instead of to liquidate, and whether the uncertainty in Chapter 11 can effect these incentives and increase the firm value ex ante.

Figure 1 shows the effect of uncertainty of Chapter 11 where we separate each uncertainty, which helps to illustrate their individual effects. First, in Figure, Panel 1, we construct equity and firm values for $g = 0$ (hands-off type with certainty) , and $g = 1$ (hands-on type with certainty), as well as $g = 0.5$, (i.e., 50%-50% percent chance for each type), where probability f for the extension of the duration of stay is varied between 0 to 1. Panel 2 depicts other comparative statics, where we hold $f = 0$, $f = 0.5$ and $f = 1$, and vary g.

The "no ambiguity" cases do not necessarily lead to the highest firm welfare. In fact, some uncertainty can be beneficial. Specifically, the certainty cases were f and g equal either 1 or 0, the outcomes are worse when compared to the case of $f = 0.5$ or $q = 0.5$. For example, in Panel 1, if $g = 1$, i.e., the judge is of hangs-on type, the firm acting in the interests of shareholders does not choose to file for Chapter 11 protection even though the firm value is higher. If hands-on judge is drawn with certainty, i.e., $g = 1$, the firm's equity value is higher if it liquidates at the liquidation boundary than if the firm chooses to enter chapter 11. The reason is that filing incurs transaction and legal expenses that are incurred by shareholders. But under the "hand-on judge", the value channeled to shareholders is likely to be zero. The "hands-on" judge will likely force the asset sales and then immediately liquidate the remaining assets, leaving no value to shareholders, but benefiting debtholders. The debtholders benefits because the assets of Division 2 that are sold are shielded from the deadweight costs of fire-sale liquidation.

On the other hand, if the firm anticipates that the judge is the "hands-off" type without uncertainty ($g = 0$), the shareholders are incentivized to file for Chapter 11 protection prematurely. The incentives to file prematurely are stronger when there is a higher probability (f) that the judge will extend the automatic stay to $T_2 = 1.5$ years. As seen on the graph, the earnings level at which the firm optimally enters for bankruptcy increases as probability f increases. Premature bankruptcy filing can hurt the firm value ex ante even more than an immediate liquidation due to higher expected financial distress costs. The shareholders tend to stay in bankruptcy without assets sales for the entire protection period, gradually cumulating distress costs and legal expenses, but still likely ending up in liquidation. Thus, in both cases, if the judge type is certain, the ex

ante outcome is not better off compared to when the firm simply liquidates without bankruptcy filing.

In comparison, if there is 50-50% chance between drawing a hands-on or a hand-off judge, shareholders may be now incetivized to enter Ch11. The firm's expected value is higher than under the liquidation, for two reasons. First, the firm postpones entering Chapter 11 until it reaches deeper distress (lower earnings), which is better that the premature filings. Second, if the hands-on judge is drawn, she will likely proceed with asset sales, which will save a part of the firms assets from costly liquidation.

A next question is which combination of the two types of uncertainty in Chapter 11 enhances the firm value relative to the case without possibility of entering Chapter 11. We construct the two dimensional area $(f \times g)$ for the combination of probabilities f and g that generate higher firm values than liquidation. Specifically, we construct the two dimensional area for which two conditions are satisfied: 1) the equityholders are willing to enter the Chapter 11 bankruptcy, $E_{Ch11}(p,d) > E_{No\_Filing}(p,d)$, and 2) the firm value is higher than that of the firm that does not have the option to enter Chapter 11 protection, i.e., $E_{Ch11}(p,d)+D_{Ch11}(p,d) > E_{No\_Filing}(p,d)+E_{No\_Filing}(p,d)$.

Finally, in Figure 2, we construct the two-dimensional area for the cases with lower prices for the firm's asset, $S = 10$, $S = 9$, and $S = 8$. Lower prices for assets implies that the asset sale boundary fall further below the liquidation boundary. As market price for the firm's asset declines, the area shrinks, implying a reduced value-enhancing effect of (uncertainty in) Chapter 11. For the $S < 7.0$, there no combination of uncertainty, $(f \times g)$ that makes the firm to pursue bankruptcy instead of liquidation, making the option to declare bankruptcy worthless for the shareholders.

In this area we also highlighted the optimal f as a function of g. In other words, for each probability of the judge type g, we find the optimal stay extension probability, f, that maximizes a total firm value. The model implies that there is a positive relation between the probabilities, where, for higher probability of hands-on judge, g, it is optimal to have a higher probability of the extension of stay, f. The intuition behind this result is the following. A higher probability of drawing a hands-on judge deters the firm's shareholders to enter Chapter 11 or incentivizes the firm to enter Chapter 11 at suboptimally low earnings. To counter this luck of incentives, the probability of extension should be longer. The result offers the implications for the optimal design of the bankruptcy courts. For example, if the court has higher fraction of hands-on judges, such court should also allow a higher probability of extension. Such court design can increase the value

of the firm that will potentially file for bankruptcy.

The Case With High Price for the firm's assets If markets offer a high price for the firm's assets, $S = 12.0 > S^*$, the firm's shareholders optimally choose to sell assets when earnings decline to p_s, if the firm has no option to enter bankruptcy. In the post-sale, the firm will continue operations with the remaining assets until the revenue declines further to p_{d_1} the liquidation boundary.[17] For $S = 12.0$, the asset sales outside the bankruptcy is the first best outcome from a firm welfare perspective. As we will show, the option to file for Chapter 11 may produce incentives for the firm's shareholders not to sell asset sale outside bankruptcy and, instead, to file for bankruptcy protection. This is a suboptimal decision from the welfare of the firm. As we will show, some uncertainty in Chapter 11 can increase such value-destroying incentives of Chapter 11 by providing stronger incentives to the firm to persue bankruptcy.

Figure 3, similar to the approach in the previous section, offers plots which separate two uncertainties of Chapter 11. First, in Panel 1, the equity and firm values are plotted for judge probability $g = 0$, $g = 1$and $g = 0.5$, while varying the probability f for the extension of the duration of stay. Figure 3, Panel 2, depicts other comparative statics, where we hold $f = 0$, $f = 0.5$ and $f = 1$, and vary g. The equityholders choose the asset sales, not bankruptcy filing if the hands-on judge is expected to be drawn with certainty, i.e., $g = 1$. If the "hands-off" type is selected with certainty ($g = 0$), the firm is incentivized to file for Chapter 11 protection for any probability of extension, instead of selling assets. For a 50-50% chance between drawing a hands-on or a hand-off judge, i.e., $g = 50\%$, shareholders files for bankruptcy (don't sell assets outside bankruptcy) if the probability of extension is high enough. As expected, the incentives to declare bankruptcy increase and the earnings at which the firm enters bankruptcy are higher, if the probability of extending the automatic stay is higher, f. For both cases of $g = 0$, and $g = 50\%$, if the firm passes up the assets sales outside bankruptcy, the firm value is lower than that if the firm sells asset. Figure 3 shows, the if the market offer a good price, the optimal design of the court is to have a relatively large probability of drawing a hands-on judge, g, which would reduce incentives for filing for Chapter 11

[17] Given that the sale price is $S = 12$, it means that the firm sells half of its production capacity ($Q_2 = 0.5$), and the sale funds can reduce the debt size (the coupon size) by at least half, i.e., if the face value (coupon size) declines from D to $D_1 = D - 12.0 < 11.5$, (from d to $d_1 < \frac{1.61}{2}$). This holds for the base case, where the debt (coupon size) declines by more then half. For the base case, the sale proceeds used to repay debt reduce the debt size from $D = 23.0$ to $D_1 = 11.0$.

without asset sales.

Figure 4 depicts the two dimensional area ($f \times g$) for the combination of f and g, at which shareholders choose not to sell asset outside bankruptcy, but file for bankruptcy protection. This combination uncertainty distorts operating decisions of the firm. Graphs present an two-dimensional area for which two conditions are satisfied: 1) the equityholders are willing to enter the bankruptcy, $E_{Ch11}(p,d) > E_{No\_Filing}(p,d)$, but 2) the firm value is lower than that when the firm sells assets outside bankruptcy, i.e., $E_{Ch11}(p,d) + D_{Ch11}(p,d) < E_{No\_Filing}(p,d) + E_{No\_Filing}(p,d)$.

The two-dimensional area are constructed for asset prices, $S = 12$, $S = 13$, $S = 14$, and $S = 15$, all higher than $S^* = 12$. For higher assets prices, the sale boundary increases further above the liquidation boundary, reducing the area in the two-dimensional space of $f \times g$ where produce suboptimal incentives for shareholders to declare bankruptcy, instead of to sell assets. For $S > 16.0$, the Chapter 11 protection gives no benefits to shareholders because there is no combination of $f \times g$ for which the Chapter 11 the equity value compared to the case with no Chapter 11 option.

The model implies the following trade-off in designing Chapter 11 during periods when the market offers good price for assets. The probability of drawing the hands-on judge should be high enough, and the probability of extension of stay should be low enough. These predictions are key for our analysis reiterating our earlier statements that during good times, Chapter 11 compounds the "reversed underinvestment" problem by further delaying asset sales and disincentivizing the firm to conduct asset sales outside bankruptcy.

4 Conclusion and Policy Implications

In this paper we shed a light on the question of how to improve the design of the Chapter 11 bankruptcy process that would strengthen the protection to debtholders and firm welfare, and at the same time mitigating the moral hazard by distressed and bankrupt firms. We offer a dynamic model which describes a game between a bankrupt company and the power and objectives of bankruptcy court, taking into account that the court can implement a mixed strategy by selecting judges that exercise the power over operating and approve the durations of the bankruptcy. The model implies that Chapter 11 process should incorporate some degree of institutional ambiguity which, if properly designed, can increases the firm value, ex ante. We concentrate on the uncertainty for the type of the bankruptcy judge that the bankrupt firm can draw at bankruptcy filing, as well as

the uncertainty whether the judge will grant the bankrupt firm an extension of the automatic stay. The model predictions offer policy implications for bankruptcy courts, that are qualitatively new to the related literature. The policy debates in the legal literature have focused on bankruptcy court uncertainty as being an ongoing problem and the transparency of courts as an ultimate solution (CITE XXX). To that extend, we suggest that the transparency about the judge composition and their judicial preferences is good as the bankrupt firms can form an expectation about the odds of the judge draw. But our results also encourage some heterogeneity of court judges with respect to the degree to which judges may interfere in the decisions of the bankrupt firm.

The "optimal ambiguity" depends on the state of the industry or the price that the market is willing to pay for the firm's assets. Specifically, ambiguous bankruptcy courts should be avoided during good times for the industry, as they can encourage premature bankruptcy filings insrtead of asset sales without bankruptcy. In good times, the bankruptcy courts should create expectations and credibly commit not to extend automatic stay and strengthen expectation that judges are more likely to take a pro-active stance during the automatic stay period. In contrast, during unfavorable times for the firm's assets, courts should promote expectation of some ambiguity, and should lower expectations about prevalence of the pro-active judges and higher expectation that the automatic stay extensions are likely.

We should stress that such policies could be difficult to implement ex ante, as it requires a delicate balance for bankruptcy courts to manage expectation about the courts in changing markets for the firms' assets. For example, theoretically, bankruptcy courts, can communicate some form of stochastic bankruptcy policy to future bankrupt firms by linking bankruptcy terms to publicly observed industry-wide variables.

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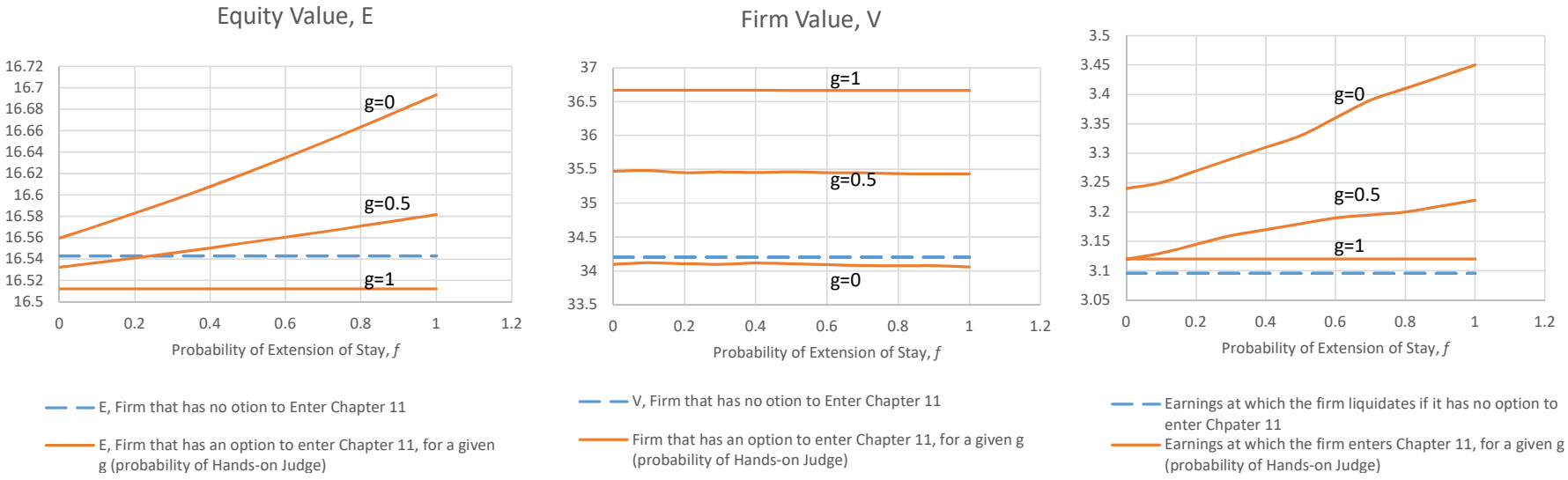
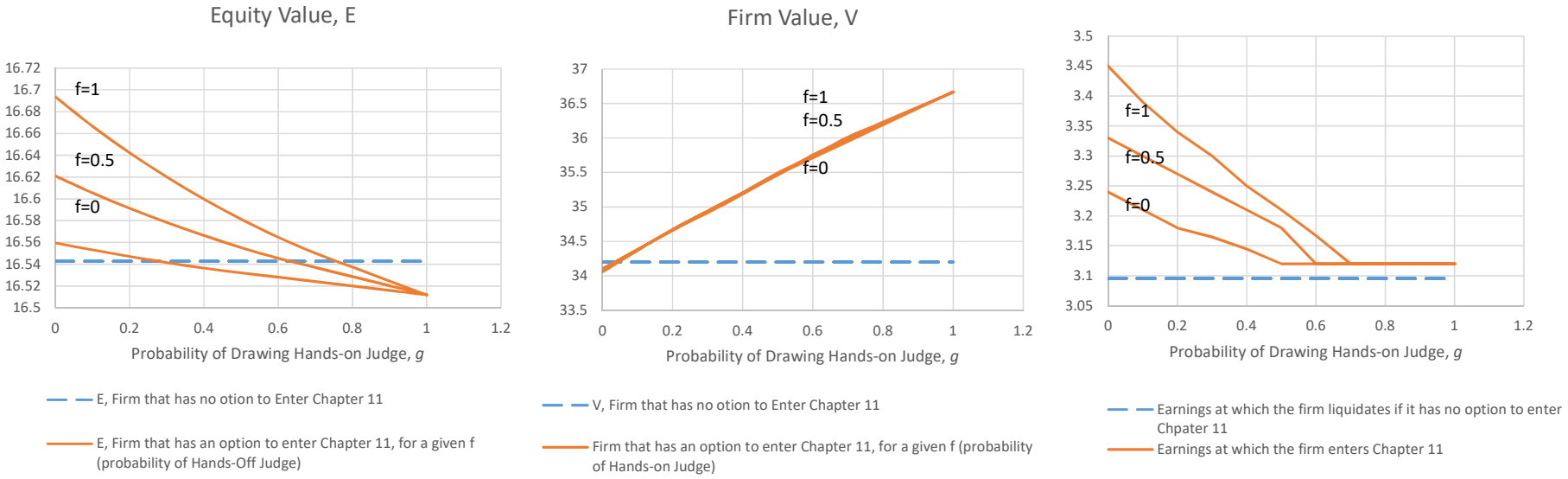
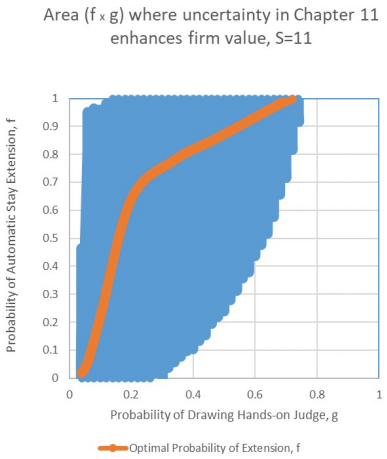
Figure 1, Panel 1. These figures present the model-generated values given the revenue p=6.0, as a function of the probability of granting an extension of stay during the Chapter 11 protection, f. The plotted values are for the two firms: 1) the firm that has no option to enter Chapter 11 protection (Dotted Blue Line), and 2) the firm that has an option to enter Chapter 11 (Red Line). For the firm that has an option to file for bankruptcy, the values are constructed for g=1 (hands-on type with certainty), and g=0 (hands-off type with certainty), as well as f=0.5, (i.,e., 50%-50% percent chance for each type). The firm filing for Chapter 11 bankruptcy faces two types of uncertainties that depends on the type of bankruptcy judge assigned to the case. First, after an initial "automatic of stay" (T=120 days) is expired, the judge can subsequently extend it for another period (for total of T=18 months). The probability that the firm draws a judge who grants an extension is f, and known to the firm, and is revealed only after filing. The second uncertainty is whether the judge is of "hands-on", drawn with probability g, or "hand-off" type with probability 1-g, which determines whether the judge exerts control over the firm's operating decisions during bankruptcy duration. Under the "hands-on" judge, the timing of the asset sales and the timing of emergence from the bankruptcy is fully under the discretion of the judge, who maximizes the total value of the firm. Hands-off judge does not control whether or when the bankrupt firm sells parts of its assets during the bankruptcy protection period and when the firm emerges from bankruptcy or liquidates. With the "hands-off " judge, the equityholders can optimally time the asset sales and the emergence from bankruptcy to maximize equityholders' value. The assignment of the bankruptcy judge type is random and the type revealed only after the firm filed for Chapter 11, and it is independent from whether the judge grants an extension.
The results are for asset sale price of S=11, for which the optimal boundary for asset sale is below the liquidation boundary. The remaining parameters are as in the base case.

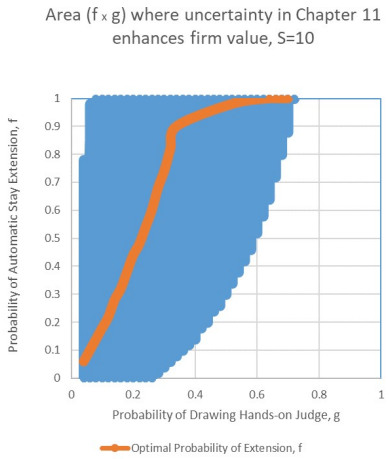


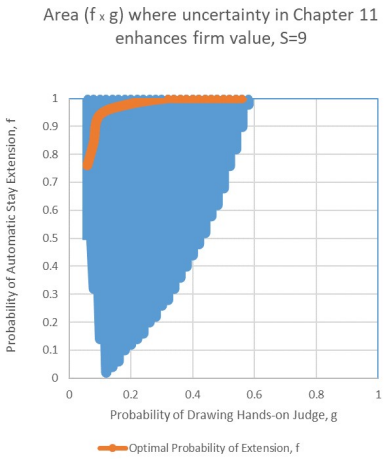
Pic 1. Panel 2. These figures present the model-generated values given the revenue of p=6.0 as a function of, g, the probability g, that the "hands-on" judge is drwn. The values are plotted for the two firms: 1) the firm that has no option to enter Chapter 11 protection (Blue Line), and 2) the firm that has an option to enter Chater 11. (Red Line). For the firm that has an option to file for bankruptcy, its values are constracted for f=0 (hands-on type with certainty), and f=1 (hands-off type with certainty), as well as f=0.5, (i.,e., 50%-50% percent chance for each type). The firm filing for Chapter 11 bankruptcy faces two types of uncertainties that depends on the type of bankruptcy judge assigned to the case. First, after an initial "automatic of stay" (T=120 days) is expired, the judge can subsequently extend it for another period (for total of T=18 months). The probability that the firm draws a judge who grants an extention is g, and known to the firm, and is revealed only after filing. The second uncertainty is whether the judge is of "hands-off" or "hand-on" type, which determines whether the judge exerts control over the firm's operating decisions during bankuptcy duration. Hands-off judge does not control whether or when the bankrupt firm sells its assets during the bankruptcy protection period and when the firm emerges from bankruptcy or liquidates. With the "hands-off " judge, the equityholders can optimally time the asset sales and the emergence from bankruptcy to maximize equityholders' value. Under the "hands-on" judge, the timing of the asset sales and the timing of emergence from the bankruptcy is fully under the discretion of the judge, who maximizes the total value of the firm. The probability of drawing the hands-off judge is f. The assigment of the bankruotcy judge type is random and the type revealed only after the firm filed for Chapter 11, and it is independent from whether the judge drants an extension.
The results are for sale price of S=11, for which the optimal boundary for asset sale is below the liquidation boundary. The values are plotted for the revenue of p=6.0 as a function of g, the probability of granting an extention of stay. The remaining parameters are as in the base case.

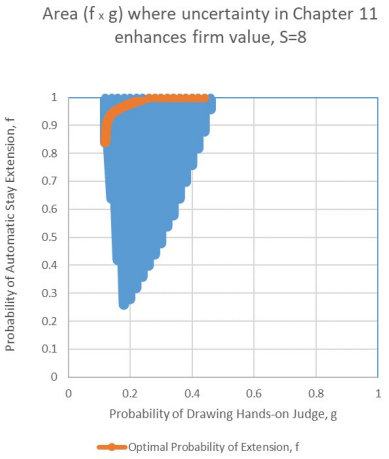


Fugure 2. The area shows the combination of the two types of uncertainties of Chapter 11, which enhances the firm value relative to the firm that has no option to enter Chapter 11, but has an option to liquidate. In this two-dimensional area (f×g) of probabilities f and g, 1) the equityholders are willing to enter the Chapter 11 bankruptcy, and 2) the firm value is higher than that of the firm that does not have the option to enter Chapter 11 protection. The curve in red is the optimal probability of extension, f, that maximizes the firm value, ex ante, for a given probability of drawing a hands-on judge, g. These areas are constructed for different market prices for assets of Division 2, varied between S=8 and S=11. These prices are lower than the critical level of S*=11.5. Thus, for these prices, the asset sale boundary is lower than the liquidation boundary, so the firm that has no option to enter Chapter 11 bankruptcy protection, liquidates without (asset sales) when the firm's earnings decline to the liquidation boundary. Other parameter values are as in the base case.

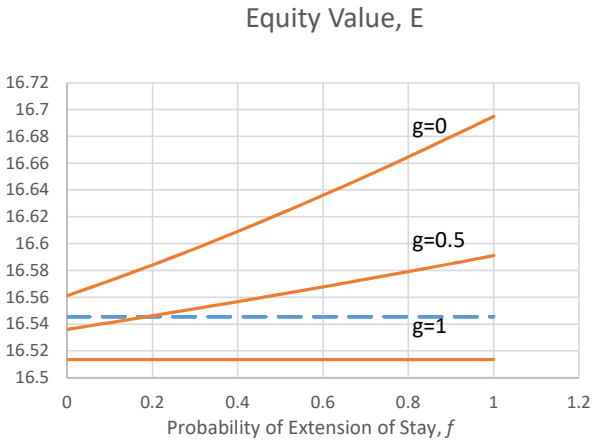


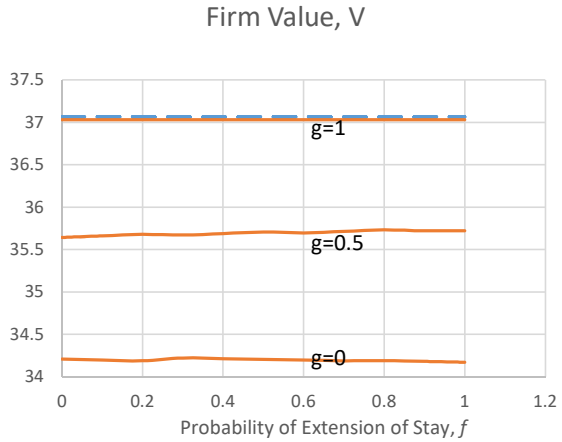


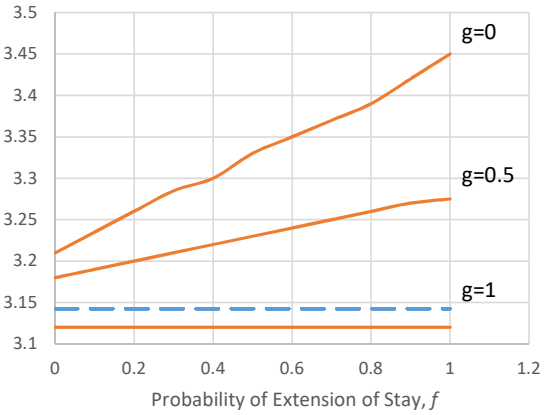


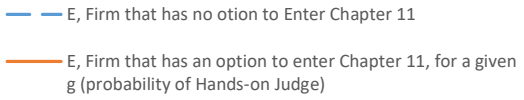


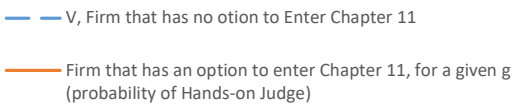
Pic 3. Panel 1. These figures present the model-generated values given the revenue p=6.0, as a function of the probability of granting an extension of stay during the Chapter 11 protection, f. The plotted values are for the two firms: 1) the firm that has no option to enter Chapter 11 protection (Dotted Blue Line), and 2) the firm that has an option to enter Chapter 11 (Red Line). For the firm that has an option to file for bankruptcy, the values are constructed for g=1 (hands-on type with certainty), and g=0 (hands-off type with certainty), as well as f=0.5, (i.,e., 50%-50% percent chance for each type). The firm filing for Chapter 11 bankruptcy faces two types of uncertainties that depends on the type of bankruptcy judge assigned to the case. First, after an initial "automatic of stay" (T=120 days) is expired, the judge can subsequently extend it for another period (for total of T=18 months). The probability that the firm draws a judge who grants an extension is f, and known to the firm, and is revealed only after filing. The second uncertainty is whether the judge is of "hands-on", drawn with probability g, or "hand-off" type with probability 1-g, which determines whether the judge exerts control over the firm's operating decisions during bankruptcy duration. Under the "hands-on" judge, the timing of the asset sales and the timing of emergence from the bankruptcy is fully under the discretion of the judge, who maximizes the total value of the firm. Hands-off judge does not control whether or when the bankrupt firm sells parts of its assets during the bankruptcy protection period and when the firm emerges from bankruptcy or liquidates. With the "hands-off " judge, the equityholders can optimally time the asset sales and the emergence from bankruptcy to maximize equityholders' value. The assignment of the bankruptcy judge type is random and the type revealed only after the firm filed for Chapter 11, and it is independent from whether the judge grants an extension.
The results are for asset sale price of S=12, for which the optimal boundary for asset sale is above the liquidation boundary, making asset sale outside bankruptcy an optimal choice for the firm welfare. The remaining parameters are as in the base case.

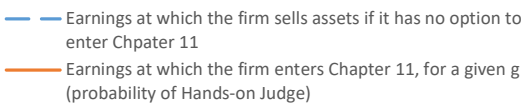




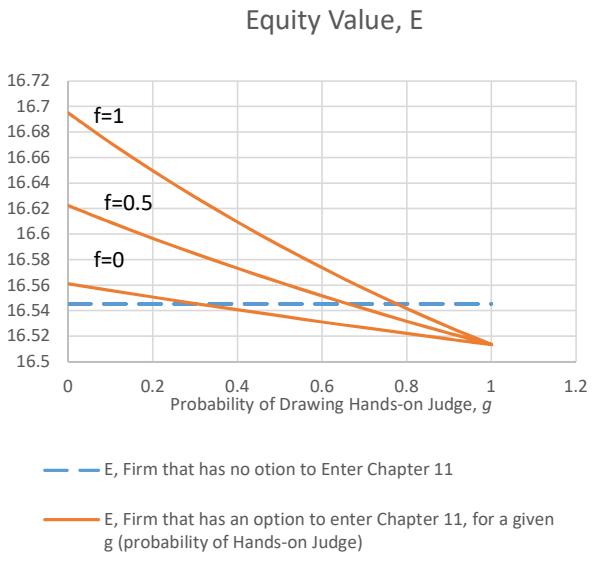


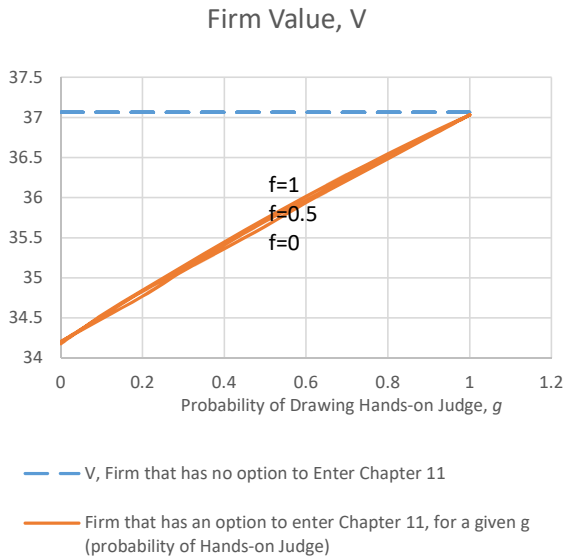






Pic 3. Panel 2. These figures present the model-generated values given the revenue p=6.0, as a function of the probability of drawing a hand-on judge, g . The plotted values are for the two firms: 1) the firm that has no option to enter Chapter 11 protection (Dotted Blue Line), and 2) the firm that has an option to enter Chapter 11. (Red Line). The firm filing for Chapter 11 bankruptcy faces two types of uncertainties that depends on the type of bankruptcy judge assigned to the case. First, after an initial "automatic of stay" (T=120 days) is expired, the judge can subsequently extend it for another period (for total of T=18 months). The probability that the firm draws a judge who grants an extension is f, and known to the firm, and is revealed only after filing. The second uncertainty is whether the judge is of "hands-on", drawn with probability g, or "hand-off" type (prob=1-g), which determines whether the judge exerts control over the firm's operating decisions during bankruptcy duration. Hands-off judge does not control whether or when the bankrupt firm sells its assets during the bankruptcy protection period and when the firm emerges from bankruptcy or liquidates. With the "hands-off " judge, the equityholders can optimally time the asset sales and the emergence from bankruptcy to maximize equityholders' value. Under the "hands-on" judge, the timing of the asset sales and the timing of emergence from the bankruptcy is fully under the discretion of the judge, who maximizes the total value of the firm. The assignment of the bankruptcy judge type is random and the type revealed only after the firm filed for Chapter 11, and it is independent from whether the judge grants an extension.
For the firm that has an option to file for bankruptcy, its equity and firm values are constructed for three cases of probability of granting an extension of stay during the Chapter 11 protection, f=1 (extension to T=1.5 years) and f=0 (no extension, T=120 days), as well as f=0.5, (i.,e., 50%-50% percent chance of extension). The results are for asset sale price of S=12, for which the optimal boundary for asset sale is above the liquidation boundary, making asset sale outside bankruptcy an optimal choice for the firm welfare. The remaining parameters are as in the base case.





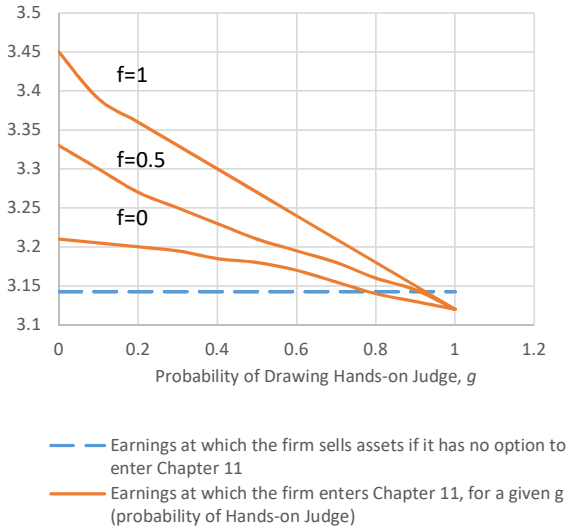


Figure 4: The area shows the combination of the two types of uncertainties of Chapter 11, the firm to foregoes asset sales and instead to file for bankruptcy, which is suboptimal from the firm value welfare. This premature filing reduces the firm value relative to the firm that has no option to enter Chapter 11, but has an option to sell assets outside bankruptcy or liquidate. For this combination of probabilities f and g, In this two-dimensional area (f×g) of probabilities f and g, 1) the equityholders are willing to enter the Chapter 11 bankruptcy, but 2) the firm value is lower than that of the firm that does not have the option to enter Chapter 11 protection. These areas are constructed for different market prices for assets of Division 2, varied between S=12 and S=15. These prices are higher than the critical level of S*=11.5. Thus, for these prices, the asset sale boundary is higher than the liquidation boundary, so the firm that has no option to enter Chapter 11 bankruptcy protection sells asset when the firm's earnings decline to the sles boundary. Other parameter values are as in the base case.