Some problems of coalition formation

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When carrying out a project, participants often need to choose their role, direction, profession, etc. To implement a project, a fixed number of agents of different professions is required. For example, to create a website, you need a programmer, designer and manager who works with the client. A football team must have a goalkeeper, an attacker and a defender. The question arises, which profession to choose in order to get the maximum income from the implementation of the project, task, game.

We use game-theoretic methods to solve problems of coalition formation. Let $N = \{1, 2, ..., n\}$ be a set of players and $M = \{1, 2, ..., m\}$ a set of professions. Given a weight matrix $W = (w_{ij})$ of size $n \times m$, where in the i-th row and j-th column the weight of player i in profession j is given. We assume that all weights are positive and the weights of any two players for the same profession are different. The player's strategy is the choice of a profession, each player chooses a single profession.

After the players have chosen their professions, in each profession they are ordered by weight (in this profession) and divided into transversals: the i-th transversal contains all the players occupying the i-th place by weight in their profession.

The payout is determined as follows. If there are less than m players in the transversal, then each of those who are in this transversal gets 0. If there are m players in the transversal, they divide 1 proportionally to their weights (in the professions they have chosen).

Our goal is to find a Nash-stable partition, that is, such a distribution of players into professions that it is not profitable for any player to change his profession.

Here are some results:

1. Let $w_{ij} = f(a_i, b_j)$, where f is monotonic function of a_i and b_j. Then there exists a Nash-stable partition. In particular, there is a stable partition for the functions $f(a_i, b_j) = a_i + b_j, f(a_i, b_j) = a_i - b_j, f(a_i, b_j) = a_i * b_j, f(a_i, b_j) = a_i/b_j$ and other.

2. Let the players be divided into 2 sets: $S_\alpha = \{\alpha_1, \alpha_2, ..., \alpha_{[\frac{n}{2}]}\}$ и $S_\beta = \{\beta_1, \beta_2, ..., \beta_{[\frac{n+1}{2}]}\}$ so that for any $i \in [1; [\frac{n}{2}]], j \in [1; [\frac{n+1}{2}]]$ performed $\alpha_{ia} > \beta_{ja}$ and $\alpha_{ib} < \beta_{jb}$. Then there exists a Nash-stable partition. In particular, there is stable partition if weights are "reversely ordered" , i.e. $w_{1A} > w_{2A} > ... > w_{nA}$, $w_{1B} < w_{2B} < ... < w_{nB}$.

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