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Model Risk for Acceptable, but Imperfect, Discrimination and Calibration in Basel PD and LGD Models

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Abstract

The Basel Internal-Ratings-Based (IRB) approach allows banks to use sufficiently good credit risk models for the daily computation of their capital adequacy ratio. However, being sufficiently good does not naturally mean being perfect. Conventionally, risk managers increase the mean probability of default (PD) and loss given default (LGD) values by some margin when developing a model. They expect that it is sufficient to offset for potential model risk. This add-on, thought to be conservative enough, gave rise to the term 'conservative margin'. The novelty of this paper is that we are the first to identify the cases when such a margin is not sufficient. The principal cause is the previously ignored requirement to "freeze" capital against the existing loans. This capital tie-up does not allow reallocating excessive capital requirements from actual non-defaults (false negatives) to unforeseen defaults (false positives). This is the first time when such a novel cause of model risk is discussed. The value the paper creates is severalfold. First, the revealed model risk has a material scale and can be part of the bank's explicit or implicit risk-taking strategy. Therefore, it should be considered by researchers, as well as by validators and supervisors. Second, the paper offers an indicator to expand the risk indicator dashboard suggested by the Basel Committee, when designing risk-reward remuneration. This is especially true of contracts with risk model developers.

Keywords: validation, IRB, Basel II, AUROC, CLAR, Loss Shortfall, Brier skill score, traffic light approach.

JEL Codes: C52, G28, G32.

1. Introduction

The Basel Framework (BCBS, 2019) preserves banks' opportunity to use their own default statistics and run their own credit risk models, and banks can compute their capital adequacy ratios with their own risk weights. This opportunity is called the Internal Ratings-Based (IRB) approach. The Basel Committee began to discuss it back in 1999 (see https://www.bis.org/publ/bcbs50.htm). It revised the calibration of IRB parameters several times within the Basel II Accord and launched the comprehensive version in 2006.

In 2020, the IRB approach is run by around half of the world's systemically important banks. Credit risk assets exceed 80% of the banking book amount, on average. Banking assets that are subject to credit risk evaluation according to the IRB methodology are equal to around 40% of world GDP. This quick snapshot demonstrates the importance of the proper measurement of credit risk and the similar importance of the IRB approach in such procedures.

An IRB model requires approval by a bank's internal validation team prior to its implementation. With the help of accumulated test use records, central bank (prudential) validation teams assess banks' IRB model adequacy prior to granting approval for use in the regular capital adequacy ratio calculation. The guiding principles for both internal and prudential validations are similar. There are qualitative and quantitative criteria (BCBS, 2005a), and there are thresholds for these criteria. Generally, validators use the traffic light approach, meaning that an evaluation score may be green, yellow or red. The green colour means very good performance for a given criterion, the yellow colour indicates worse but still acceptable levels, and the red colour signifies the presence of material deficiencies, rule violations, fatal problems, etc. The red colour is definitely unacceptable. However, the green colour is not a guarantee of perfect performance: it represents good or very good, but not necessarily ideal, results. For instance, a green rating for discriminatory power in LGD models might imply that the corresponding cumulative LGD accuracy ratio (CLAR) test yields values of around 80%, that is above the 75% threshold (Maarse, 2012, pp. 53, 76-77). Discriminating at the 80% level is high enough by industry standards, although perfect discrimination would equal 100%. This means that we are still somewhat imprecisely measuring portfolio credit risk. Simply stated, there is model risk, meaning that we may suffer losses from models that do not perfectly mimic the real-world data generation process. This may happen even when the models are in the green zone. Unquestionably, red-zone models imply higher model risk than green-zone ones. Thus, the research question naturally comes to mind: Do green-zone models (i.e., of acceptable, but not perfect, quality) imply any material risk underestimation? If such underestimation occurs, we wish to use a mark-up for the model output to cover this underestimation. This is why we model the amount of the mark-up to the total estimate of credit risk in the IRB models when the underlying PD and LGD models may be not perfect, but are acceptable, i.e., in the green or yellow zones, by the tests of discriminatory power and calibration.

Whereas the focus of our paper might seem too technical at first sight, it has some broader implications. First, banks seek to cut down costs by opting out of further IRB model improvement. Such decisions might be solely driven by financial cost optimisation or result from strategic decisions made with regard to the amount of risk-taking by a bank. When such decisions are deliberately made, risk-taking is explicit. However, in some instances a bank's top-management does not recognise the additional model risk that it takes. Then there is implicit risk-taking. Here we are not searching for the determinants whether risk-taking is explicit or implicit. Our intent is to demonstrate the scale of the model risk that arises from previously non-considered IRB features of capital "freeze". Second, by evaluating the model risk scale, the paper offers the tools for the bank top and risk management, for its auditors (external and internal) and its supervisors to revise remuneration schemes to order to incentivise model developers to permanently improve the IRB models' accuracy and discriminatory power.

The paper is structured in the following way. Section 2 is a brief literature review outlining how the discussion of model risk has evolved. Section 3 presents the methodology and demonstrates the simulated data, with an explanation of the source of model risk in the case of borrower default. Section 4 offers the major findings in statistical tables that practitioners may readily use for PD and LGD model validation; furthermore, this paper offers mark-ups to total estimates of credit risk.

2. Literature review

A spike in the attention paid to model risk occurred after the global financial crisis of 2007–09. This was primarily related to the inadequate risk assessment of securitisation transactions and collateralised debt obligation (CDO) pricing. Copulas have been used as one candidate for exacerbated model risk (MacKenzie & Spears, 2014). As a result, the US Federal Reserve adopted general principles for looking at model risk (FRS, 2011). However, these guidelines are principle-based and include no concrete quantitative thresholds. Several years later, the US Federal Deposit Insurance Corporation lent its support to the implementation of these guidelines (FDIC, 2017). Its focus was on third-party providers considering that their 'black box' solutions may also lead to increases in model risk.

The discussion around the presence of model risk in the IRB approach is more complex. There are two sides to the coin, one more academic and one more regulatory.

First, model risk has been the subject matter of several papers. These are Loffler (2003), Tarashev and Zhu (2008), and Tarashev (2010). They all assume that there is uncertainty in the risk parameters, including PD and LGD. Loffler (2003) focuses more on credit ratings, while Tarashev and Zhu (2008) and Tarashev (2010) consider applications for credit risk estimates. However, they do not notice the rigidity of predicted capital requirements. This means that if risk managers believe in their models — often after a series of rigorous validation procedures — they expect that more capital is needed for less creditworthy borrowers. However, when the models do not perfectly discriminate, risk managers think that bad borrowers are not as bad as they actually are and do not require as much capital as is in fact needed. When those borrowers default, capital plummets, but the expectedly bad borrowers still require no less capital than before. Neither Loffler (2003), nor Tarashev and Zhu (2008), nor Tarashev (2010) consider such rigidity. This paper closes the gap.

Tikhonov et al. (2021) try to reveal the link between bank profit and model risk. They claim that predicted defaults that are actually non-defaults (they call this a type I error assuming a null hypothesis of no default) do not enable the scoring of potential profit for a bank, while granting a loan to an actual, but unforeseen defaulter (they call this a type II error) brings direct losses to a bank. Thus, they argue that the model risk is the sum of the direct losses and the unearned profit. Whereas they look at the stage of loan approval, this paper takes another step: it examines already approved loans and looks for the model risk associated with them. It demonstrates that the approach of Tikhonov et al. (2021) does not consider the rigidity of the capital requirements for the predicted defaults in the portfolio of approved loans. This means that the actual model risk is larger, as this paper will show. In some ways, this finding coincides with that of Kupiec (2009), who argues that the advanced IRB approach cannot properly mimic default rate data patterns and as a result significantly underestimates credit risks.

Second, in discussing regulatory guidelines on the treatment of model risk, the European Banking Federation considers that the standardised approach has more model risk than IRB (EBF, 2015). They stress the discrete predefined values in the standardised approach, which imply cliff effects. However, this does not contradict the possible presence of model risk in the IRB framework, although model risk may be lower in the IRB approach. The Basel Committee itself admits that model risk may originate from the use of the wrong methodology or the presence of unobserved factors (BCBS, 2019b, pp. par. 50.10, 50.13). The committee also recommends validation adjustments to account for model risk, though the guidelines offer no concrete statistical estimates. Ermolova et al. (2019) attempt to provide such estimates. In that paper, however, we limit ourselves to a single parameter, the probability of default (PD), which narrows the scope of application of our findings to Foundation IRB for corporates. Otherwise, for the advanced approach and for retail exposures, the validator must know the model risk implications from the two parameters — PD and LGD. When both are considered, their correlation, or PLC, should also be considered. This means that the existing literature has omitted consideration of the model risk connected with the LGD. This paper addresses that problem.

The quantitative tests for the validation of PD and LGD models focus mostly on discriminatory power and calibration. The former is similar to the notion of correlation, that is, how well the model distinguishes high values from low ones (defaults from non-defaults, etc.). The latter shows the accuracy of the mean parameter levels. In other words, it is a comparison of the predictions for the model portfolio to the central tendencies. Bankers often do add, or regulators advise adding, a conservative margin to offset for some unobserved determinants, including PLC. This paper uses the thresholds for the four validation tests from the BCBS (2005a), Pomazanov (2016, p. 54), and Kramer and Neumarker (2019), for the PD models, and from Maarse (2012, pp. 53, 76-77), Bituytsky et al. (2013, p. 55), and Vujnović, et al. (2016, p. 469) for the LGD models. See Table 1.

Table 1. Discriminatory power validation thresholds for PD and LGD models

In percentage points	AUROC for PD		CLAR for LGD	
Model status	**min**	**max**	**min**	**max**
Unacceptable (red)	0	50	50	60
Satisfactory (yellow)	50	70	60	75
Good (green)	70	100	75	100

Of particular concern is the good (green) domain, when the AUROC for PD models is above 70% and the CLAR for LGD models exceeds 75% in Table 1. There is plenty of room for the models to be less than perfect. In particular, models close to these boundaries may bring significant model risk even with a sufficiently conservative margin (e.g. 5 pp) added to the calibrated central tendency levels. This paper does not touch upon the issue of point and interval estimates for these model features raised by Engelmann et al. (2003), that being a separate research extension outside its scope.

Bank risk-taking has already been studied with a particular focus on implications for monetary policy (Cecchetti & Li, 2005), (Borio & Zhu, 2008), (Bruno & Shin, 2012), (Dell'Ariccia, 2013), (Jimenez, Ongena, Peydro, & Saurina, 2014), (Tressel & Verdier, 2014), (Malovaná, Kolcunová, & Brož, 2019). These papers are focused more on the asset side of bank activities. Their principal finding is that soft monetary policy tends to promote excessive risk-taking. On the contrary, several other authors (Schoors, Semenova, & Zubanov, 2019) draw attention to the liability side of banking books. They associate higher risk-taking with a lower capital adequacy ratio (CAR) or a higher non-performing loans ratio (NPL). The banks that have taken higher risks tend to offer higher deposit rates to more actively raise household funds. However, the mentioned papers do not consider the implications of model risk.

Below we will demonstrate that model risk might be material and reach up to 9% of the total predicted credit risk amount. This is valid for the following set of parameters: an LGD CLAR of 80%, a PD AUROC of 75%; the mean PD and LGD values are 5 pp above their central tendencies; PLC equals 50%. However, we are not discussing whether model risk-taking by a bank is deliberate or random, i.e., whether it is explicit or implicit. Our objective is to draw attention to the importance of considering the novel model risk feature in both academic literature and day-to-day business of a bank, validators, auditors and supervisors.

Moreover, the paper findings are useful for the design of risk-reward systems. The optimal design of a remuneration system has attracted the attention of academicians for at least forty years, see (Grossman & Hart, 1983), (Bull, 1987), (Mirrlees, 1999), (Gibbons, 2005). Their key message is that such a design should focus on observed indicators of output, rather than on unobserved proxies of applied effort. The Basel Committee has recently published the description of the best practices in risk remuneration (BCBS, 2011). Essentially, it proposes to differentiate between bonus schemes for risk-takers and risk-controllers. The remuneration amount should include fixed and variable components. The latter one should be broken down into instant and deferred parts. The triggers for the variable and deferred amounts should be chosen out of the particular list of observable risk indicators. Our paper suggests the use of the model risk proxy as an indicator that may well fit the risk dashboard described by the Basel Committee.

3. Methodology and simulated data

One may think that setting an overly conservative margin would easily offset any model risk within the IRB approach. However, this is not always true. High risk estimates may be overly conservatively assigned to the wrong borrowers. This happens when the discriminatory power is imperfect, as it always is in real life. As a result, a low amount of capital (in terms of both expected loss (EL) and unexpected loss (UL)) is provisioned for bad borrowers, but an excessive amount is provisioned for creditworthy ones. The bad borrowers default more often, however, and they bring higher losses after recoveries than the good ones. When the former bad borrowers default, close to 100% of capital must be allocated to their exposures. At the same time, the high capital requirements for the good borrowers remain in place in order to follow the overly conservative requirements. Capital cannot be reallocated from the good borrowers, as the models are still trusted. Thus, there is a double capital cost. It is falsely high for the good borrowers, and it is actually high for the bad ones when they default. This means that, most often, since the PD and LGD are imperfectly discriminatory, there are model risk implications from the IRB approach. This is the core element of the methodology used here. This concept can be illustrated using the conventional type I and II error matrix from Table 2.

Table 2. Capital requirement in the presence of model risk

Defaults		Predicted		TOTAL
		ND	D	
True	ND	K_{11}	K_{12}	$K_{1\bullet}$
	D	K_{21}	K_{22}	$K_{2\bullet}$
TOTAL		$K_{\bullet 1}$	$K_{\bullet 2}$	$K_{\bullet\bullet}$

Note: ND – non-defaults; D – defaults; K_{kl} - capital requirements for the observations in the k-th row and l-th column; $K_{k\bullet}$ - total capital requirements for the observations in the k-th row; $K_{\bullet l}$ - total capital requirements for the observations in the l-th column.

Consider the null hypothesis that the borrower is going to default. Then the type I error is the number of true (actual) defaults that were not foreseen. The corresponding capital requirement is K_{21}. The type II error is the number of non-defaults that were mistakenly assigned to default status. The corresponding capital requirement is K_{12}. The total capital requirement for the portfolio is $K_{\bullet\bullet}$.

Type I error in this example means that the capital requirements of K_{21} are lower than needed, while the type II error implies that K_{12} is higher than sufficient. One may then suppose that the surplus of K_{12} may at least in some part offset the deficit in K_{21}. The model risk seems to be equal to the following:

$$\max\left\{\tilde{K}_{21} - K_{21} + \left(\tilde{K}_{12} - K_{12}\right); 0\right\}, \quad [1]$$

where $\tilde{K}_{ij}$ is the full capital requirement sufficient for the i-th row and j-th column of Table 2.

However, the viewpoint of [1] has a material shortcoming: it does not account for the rigidity of capital requirements. Consider a simple case in which actual defaults take place. The bank must increase provisions and deduct the amount $\left(\tilde{K}_{21} - K_{21}\right)$ from capital. However, the bank cannot take the capital from other allocations like $\left(\tilde{K}_{12} - K_{12}\right)$, as the bank does not know $\tilde{K}_{12}$. The bank's risk managers believe in their models and believe that the capital requirements are higher than $\tilde{K}_{12}$ and equal to the amount of K_{12} until such loans are fully redeemed. This means that the bank is stuck with its existing capital requirements $K_{\bullet\bullet}$, though it actually requires $\left[K_{\bullet\bullet}+\left(\tilde{K}_{21}-K_{21}\right)\right]$. Thus, the model risk is larger than in [1] and is equal only to the part not provisioned for the unforeseen defaults, i.e.,

$$\max\left\{\tilde{K}_{21}-K_{21};0\right\}=\tilde{K}_{21}-K_{21} \quad [2]$$

Thus, the necessary total risk mark-up may be defined as follows:

$$\frac{\tilde{K}_{21}-K_{21}}{K_{\bullet\bullet}} \quad [3]$$

The important point — principally different from the approach of Loffler (2003), Tarashev and Zhu (2008), and Tarashev (2010) — is the denominator. It is the existing capital requirement originating from the regulatory regime in place. For instance, part of the capital requirements may be derived using the IRB approach.

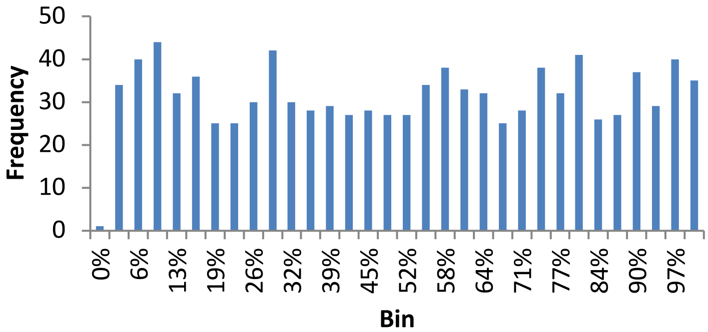


Figure 1. Univariate distribution for PD

Simulated data are used to derive the total risk mark-up, using an artificial portfolio of one thousand equally sized loans. A univariate distribution of PD as in Figure 1 is used. This enables the management of the default rate. When the interest is in simulating a given default rate DR, defaults are assigned to observations by the binomial rule. Default is present when the default probability is higher than (1-DR). If another sort of empirical-like PD distribution were used, it would hardly be possible to observe extreme PD values above the (1-DR) threshold. Defaults should be assigned to a DR proportion of total observations, but then it may be possible to have a default with an actual PD of, say, 20% with a DR of 15%. That is why the univariate PD distribution from Figure 1 is used: in order to be able to transparently and explicitly assign defaults.

Beta distribution is used for the LGD, as in Figure 2, thus imitating the frequently mentioned U-shaped distribution for the LGD (see Ozdemir and Miu (2009, p. 18), Arsova et al. (2011, p. 3), and Yao et al. (2014, p. 2)).

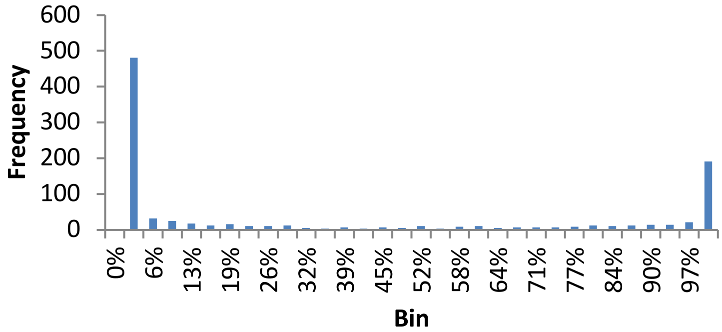
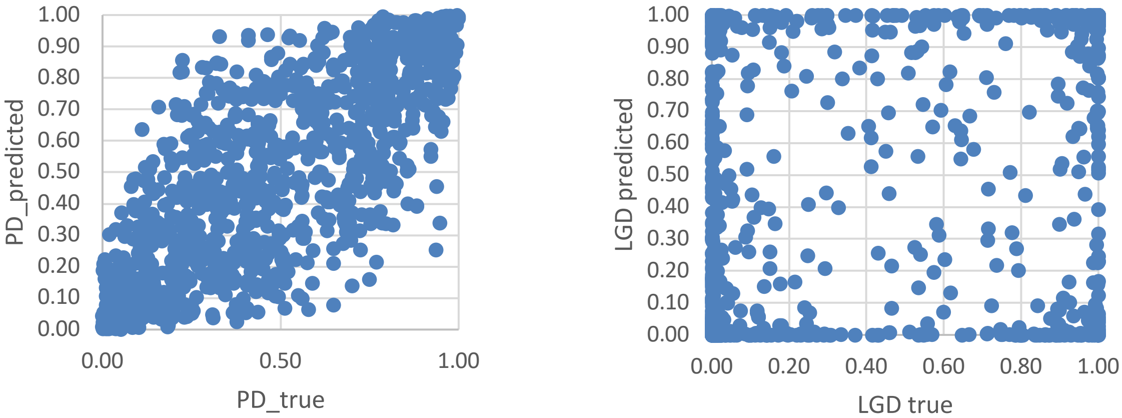


Figure 2. Beta distribution to obtain U-shaped LGD (mean LGD = 33%)

The correlation of the actual (true) and predicted PD and LGD values are varied, respectively, as shown in Figure 3. As both IRB parameters — PD and LGD — are dealt with, the various dependencies between them must be considered. Such dependency is known as the PD-LGD correlation (PLC). It is illustrated in Figure 4.



(a) PD: correlation is +75%. (b) LGD: correlation is +50%.

Figure 3. Modelling of discriminatory power via the correlation of actual and predicted values

To simulate correlated random variables, the conventional approach to simulating correlated Gaussian variables from [1] is used. If the wish is to model two random variables X_i and Z correlated with correlation coefficient ρ, there must be an auxiliary random variable Y_i. In fact, Vasicek (1987; 2002) uses the very same approach (Vasicek, 1987) and (Vasicek, 2002), with Z a systemic risk factor; X_i being a particular borrower's asset returns, and ρ^2 being the asset correlation:

$$X_i = \rho \cdot Z + Y_i \cdot \sqrt{1-\rho^2} \quad . \tag{4}$$

Five values are used for the actual and predicted PD correlations (0%; 25%; 50%; 75%; 100%), four values for the actual and predicted LGD values (0%; 20%; 90%; 100%), three values for the PD-LGD correlations (PLC: 0%; 25%; 50%), and three values of accuracy (precise, no deviation in mean; 5 pp above the mean and 5 pp below it). The value of PLC equal to 50% corresponds to the findings of (Meng, et al., 2010) for US data.

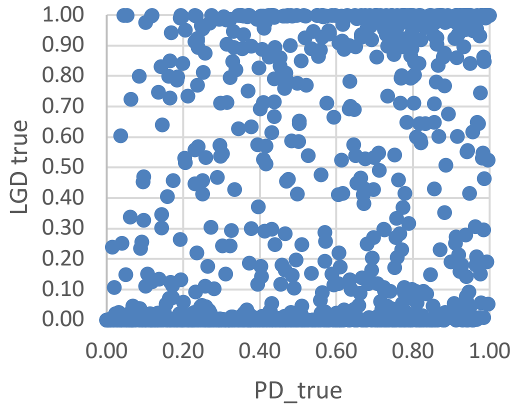
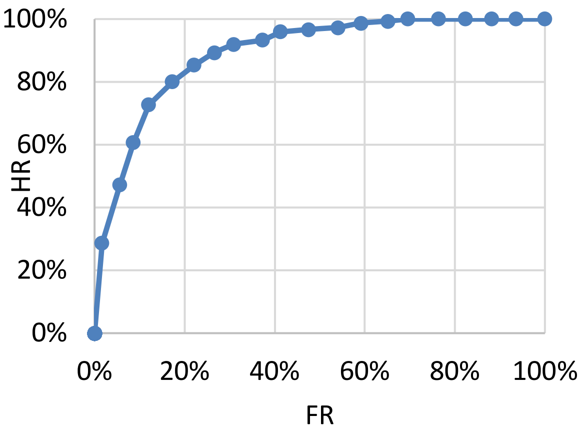
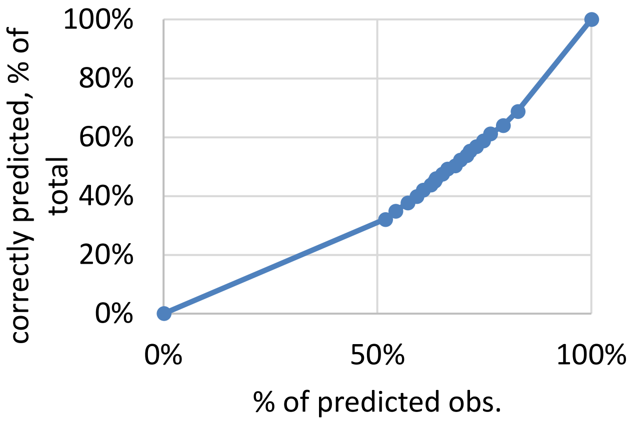


Figure 4. This paper looks at various PD-LGD correlation (PLC) values (here it is +50%)

This paper considers the baseline case with a mean actual PD of 15% and a mean actual LGD of 35%. The latter is the benchmark value available for Foundation IRB, given that collateral adjustments are in place. Overall, 675 scenarios are run.



(a) AUC = 89% for correlation of actual and true PD values of +75%.



(b) CLAR = 77% for correlation of actual and true LGD values of +50%.

Figure 5. Accuracy ratio is used to present output in statistical tables below

The discriminatory power of PD and LGD models is measured with the accuracy ratio (area under the ROC curve, AUC) and the cumulative LGD accuracy ratio (CLAR). An illustration of the test is presented in Figure 5. The ROC curve is presented using 20 points, and 100 points are used to compute the AUC. The computation of CLAR uses 20 groups.

The calibration is the difference in the mean values of the true and predicted PD and LGD values. The Brier Score (BS) and a normalised Brier Skill Score (BSS) are considered but not presented in the output tables, because they depend on the PD and LGD values, making them incomparable for different scenarios and making it impossible to readily apply the tables developed in the day-to-day work of risk managers, validators and auditors.

The IRB risk-weighting formula for general corporate loans is used and the total capital requirement K for a loan with given parameters PD and LGD is computed (BCBS, 2019, pp. CRE, par. 31) (see [1]). The $PD \cdot LGD$ product is intentionally not deducted, as the total risk estimate not limited to the unexpected loss component is needed. An equal loan maturity M of 2.5 years is assumed. The total credit risk is computed as the amount of risk-weighted assets from the CAR denominator without multiplying by 12.5 and the amount of expected losses (subtracted from both the numerator and denominator). The total credit risk is estimated for the actual and predicted data for each artificial loan.

$$K = LGD \cdot \left(\frac{N^{-1}(PD) + N^{-1}(0.999) \cdot \sqrt{1-\rho^2}}{\sqrt{1-\rho^2}} \right) \cdot \left(\frac{1 + (M - 2.5) \cdot b}{1 - 1.5 \cdot b} \right),$$
$$b = \left(0.11852 - 0.05478 \cdot \ln(PD)\right)^2,$$
$$\rho^2 = 0.12 \cdot \frac{1-\exp(-50 \cdot PD)}{1-\exp(-50)} + 0.24 \cdot \left[1 - \frac{1-\exp(-50 \cdot PD)}{1-\exp(-50)} \right]. \qquad [5]$$

To compute its prudential equivalent, the predicted risk amounts are substituted with the actual values for true defaults. As discussed above, it is necessary to preserve the capital requirement according to the model predictions for the non-defaults. The models are trusted, and it is expected that high-risk values may occur up until loan maturity if so predicted. The predictions are substituted with the actual values as it is necessary to instantaneously register losses when a borrower stops paying, disregarding how low the credit risk for this is predicted to be. When it is said that a borrower has stopped paying, it is assumed that it has a payment overdue by more than 90 days or that certain UTP criteria have been triggered. Finally, there are two values for the total credit risk: the predicted value and the prudential value (the latter is the predicted value adjusted for the defaults realised). The total risk mark-up is defined as the ratio of the prudential estimate to the predicted value. From a conservative point of view, the mark-up value is limited to zero from below.

4. Findings

4.1. Single scenario demonstration

Table 3 presents the logic for a single scenario for a better understanding of how the general output is arrived at. Take a loan portfolio with true values of PD = 15% and LGD = 31%. The latter corresponds to a Beta (0.10, 0.20) distribution. Let there be more conservative risk parameter predictions, with mean PD = 20% (+5 pp more conservative) and mean LGD = 34% (+3 pp more conservative; this is a Beta (0.12, 0.20) distribution). This yields a Brier Score of +23%. The PD-LGD correlation (PLC) is +50%. Let the model be quite good, though not perfect, in discrimination. The correlation of actual to predicted PD values is +75%. This corresponds to an AUROC of +89%. It falls in the green zone for validation purposes.

Table 3. Demonstration of a single model risk scenario

(a) INPUT		Calibration		
	Discrimination	dif (F-A)	BS	BSS
PD	89%	5%	23%	-75%
LGD	77%	3%	23%	

(b) OUTCOME	RWA	EL + UL
actual (A)	143.63	296.23
forecasted (F)	108.19	317.81
prud (frcst + actual)	216.83	327.83
(pru / frcst)-1	100.4%	3.2%
mark-up	100.4%	3.2%

The actual-to-predicted LGD values are less correlated, with a coefficient of +50%. Nevertheless, the corresponding CLAR is +77%, and it is also within the green validation zone. To sum up, the PD and LGD models are more conservative (by +5 and +3 pp respectively). They do not perfectly discriminate, but still successfully pass validation without a need for further improvement.

The portfolio under consideration has a total credit risk of 296 units (EL + UL), with the unexpected part (RWA) equal to 144 units. The predicted (forecasted, F; frcst) risk amount is 317.81 (318 if rounded). The prudential (pru) risk estimate considers full capital consumption for defaulted loans. If a loan is going to default, the forecasted credit risk amount is used, otherwise the actual (provisioned) capital requirement is used. Thus, the model risk from the setting in [2] leads to a credit risk estimate of 328 units. This is larger than the forecasted amount of 318. This means that the model risk add-on is roughly 10 units, or 3.2% of the forecasted total amount of risk. This might be a statistical error. However, most of the underestimation comes from the unexpected loss part. The prudential estimate for this is 217, while that forecasted by the models is only 108. Thus, the model risk in the unexpected part is 109 units, or close to 100% of the forecasted amount. The total risk mark-up equals to 3.2%.

4.2. General patterns

To summarise the key findings:

1) The acceptable PD and LGD models are examined. They are in the yellow and green zones by the criteria for discriminatory power. Conventionally, all of them are acceptable from the point of view of validation. However, the underestimation of credit risk may reach around 20% of the predicted amount for particular combinations of parameters. For instance, see LGD CLAR of 71% and PD AUROC of 50% given a PLC of 50% in Figure 6.

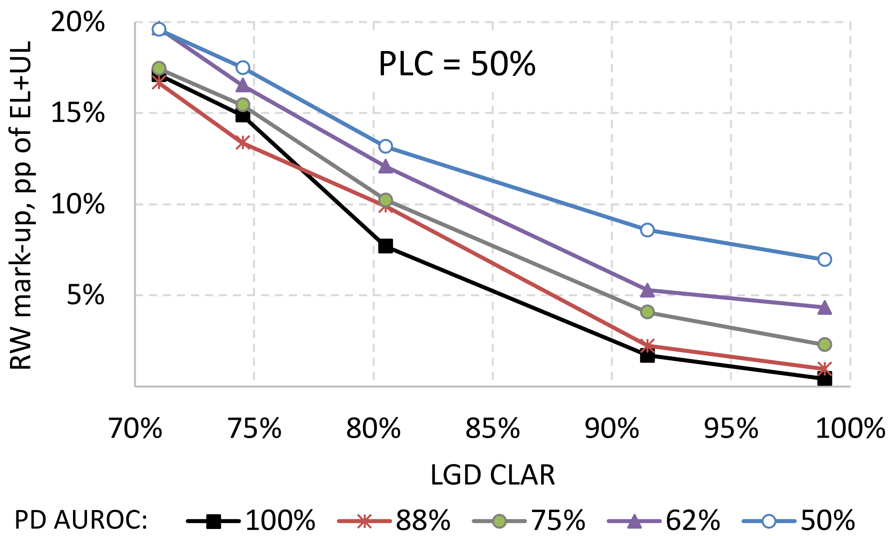


Figure 6. Model risk mark-up sensitivity analysis by discriminatory power of models

2) The higher the PD-LGD correlation is, the higher the model risk mark-ups are.
3) Introducing a 5-pp conservative margin is not always sufficient, though the add-on may not be large (up to a couple of percentage points). However, the models less conservative in the mean may imply non-proportionally higher add-ons. For instance, -5% in PD with -5% in LGD requires an 11.5% mark-up on average for PLC =50% (though we could have expected it to be equal to 10 pp. as a sum of 5$ for PD and 5% for LGD). See Figure 7.

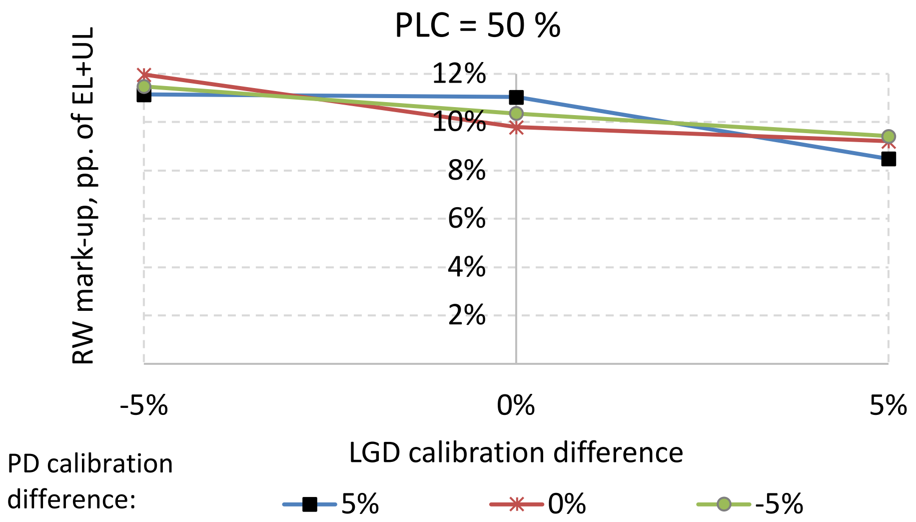


Figure 7. Model risk mark-up sensitivity analysis by model calibration

Note: Calibr_dif is the difference between the mean value of the model and the central tendency.

4) Each percentage point of underestimation in LGD is more 'costly' than each percentage point of underestimation in PD. For instance, a PD margin of +5% and an LGD margin of -5% imply a mark-up in the range of 3.2%–11.2%, depending on PLC.
5) The green-zone indicator for the quality of the PD and LGD models does not always imply proper risk assessment. It would be useful to add the quantitative estimate of model risk to validation reports. Interested parties (including regulators) may wish to consider model risk when deciding whether to allow a bank to use the IRB approach or when deciding whether a bank may need a mark-up in the use of IRB.

5. Conclusion and discussion

Model risk is still on credit risk managers' agenda. The Basel Framework prescribes the development of PD and LGD models for those banks wishing to employ the IRB approach. While it is only natural to accept that that real life is not perfect, it is desirable to offer a solution for offsetting the arising model risk for imperfect models. Importantly, such imperfections cannot be considered solely by making models more conservative from a calibration standpoint. They also need to be more discriminative. If such improvements are not feasible, this paper has justified amounts of total credit risk add-on to account for such imperfections — such that banks may preserve their solvability.

The paper has demonstrated the amount of credit risk underestimation due to model risk in the PD and LGD models within the IRB framework. It has dealt with total risk and is not intended to discuss the proper allocation of the mark-up among the EL and UL, which may become a separate research topic. However, it should be noted that there might be two extremes. From one side, a local regulator may prescribe a conservative deduction of the entire mark-up as the provision amount (this would be an EL equivalent). From the other side, the milder approach is to multiply it by 12.5 and allocate it as the add-on to the amount of risk-weighted assets or a risk-weight mark-up (this would be an UL equivalent). This latter approach is more typical of macroprudential policy tools.

The proposed IRB model risk add-on is useful for incentivising model developers to improve their models. The possibility of allocating the add-on in full or in part gives risk and top managers support for investing in data mining, data quality improvement, and model improvement. Even if a regulator does not wish to adopt the justified model risk add-ons, bank managers may use them internally as a KPI for economic capital and for model developers, model validators and auditors (both internal and external). Ultimately, credit models should be oriented toward higher bank resiliency and higher overall financial stability.

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