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RESERVES IN TARGETED ADMISSIONS: MECHANISM DESIGN APPROACH

SERGEI DIAGILEV AND ALEXANDER NESTEROV

Abstract. We introduce a new problem of assigning targeted seats in college admissions. Targeted seats are reserved for students that precommit to be employed by particular firms after graduation. We model firms as strategic agents and study how to design reserves in the spirit of dur2018reserve. When firms use the programs' ranking of students and trim it according to their preferences, there exist a reserve design that is stable, strategy-proof for students and has good properties for firms. Otherwise, when firms rank students differently, the reserve design with these properties becomes impossible. Our results suggest how to let firms express their preferences over students and how to design reserves for targeted seats.

1. Introduction

Targeted admission is a term for a joint three-sided agreement between a student, a study program, and a firm, that would hire the student after a successful completing of the study program. As such, targeted admission is a theoretically useful way to coordinate education and labor market: the students get guaranteed employment after graduation, the firms get trained employees, and programs become matching and training platforms and can get financing from the firms. Finally, the state gets a more predictable and reliable human resources development en large, and the state is a key player in this game. For example, in Russia in 2024, out of 600 thousand seats financed by the federal budget, 150 thousand seats are reserved for targeted admission. Similar practices exist in China and Brazil.

However, designing a good targeted admission system is not easy as it requires taking care of three sides, and the side of firms is completely independent. In Russia, the current system is very restrictive and impedes the normal admission process. Specifically, in order to compete for a targeted seat, each student can choose only one program and only one firm, which is not at all in the spirit of admission for general seats. Moreover, for each student the targeted seat at a program will be treated as more preferred compared to a general seat at the program, which might go against the true preferences of the student and thus make him reluctant to apply for a targeted seat at all.

This paper contributes into designing a good targeted admission system that accounts for these and other issues.

We propose a new model of allocating seats reserved for targeted admission with different firms and study how to adjust and reallocate these reserves based on the students' demand. We find that when firms' preferences are homogeneous (i.e. when they are based on the students' grades same as the priorities of the program, but possibly with some trimming due to acceptability issues), then there exist a reserve allocation rule with desirable properties. This rule is stable (at a given program, no student-firm pair can block the final matching of seats) and strategy-proof for students (at a given program, for each student it is a dominant strategy to report preferences over firms truthfully). The scope of strategic manipulations for firms is also very limited.

2. Model

We adjust the standard college admission model to the three-sided student-school-firm matching problem, though we mostly focus on the case with one school.

2.1. **Setup.** The matching problem presented in this paper includes three types of agents. There is a finite set of students I, a finite set of firms F, and a school S. The school has a total capacity, which we will denote by Q. This capacity is to be split between two types of school places: those for general admission and those for targeted admission. Obtaining a general admission place does not imply additional terms for students. In contrast, obtaining a targeted admission place is only possible in conjunction with the signing of a student-firm contract. Such a contract usually includes mandatory employment with a firm for several years after graduation. The school S reserves R places for targeted admission.

Firms also have restrictions on the total number of students with whom they are ready to sign a contract. We will denote the capacity of each firm by Q_f. To incorporate general admission places into our model in a simple way, we introduce a dummy firm f_0. For a student, signing a contract with the firm f_0 would simply mean obtaining a general admission place at the school S. The capacity of the dummy firm Q_{f_0} is equal to the capacity of the school, since there might be insufficient demand for targeted places.

The agents in our model have preferences for each other. We will assume that they list only acceptable for them options in their preferences. There is one additional restriction on firms' preferences: only students acceptable to the school can be acceptable to firms. Students have preferences $P_I = (P_i)_{i \in I}$ over the set of firms F. The school has preferences P over the set of students I. Firms have preferences $P_F = (P_f)_{f \in F}$ over the set of students I. Preferences of firm f_0 always coincide with preferences of the school.

Our model is based on the *matching with contracts* model by Hatfield and Milgrom (2005). In our setting, each contract $x = (i, S, f)$ includes some student i (x_I), the school S (x_S), and some firm f (x_F). $X = I \times S \times F$ denotes the set of all possible contracts. For each agent, preferences over contracts can be easily retrieved from the agent's initial preferences (P_i, P, or P_f).

2.2. **Mechanism.** Let us formally define the matching mechanism φ. It resembles the Gale-Shapley algorithm and consists of multiple steps.

First, we need to introduce some additional notation. A^k denotes the set of contracts which the school S temporarily accepted by the end of step k. For each firm $f \in F$, A^k_f denotes the set of contracts which f temporarily accepted by the end of step k. There is also a set $A^k_{-f} = A^k \setminus A^k_f$.

Second, we need to introduce a choice function C. This choice function selects $N - 1$ highest ranked contracts according to P from any set of contracts with some cardinality $N \geq 2$.

The mechanism φ takes sets I and F; the school S, its capacity Q and reserve size R; preferences P_I, P, P_F; and a vector $Q_F = (Q_{f_0}, Q_{f_1}, \ldots)$ with capacities of firms. Then it produces an allocation μ as follows:

- **Step 1.** Some student $i \in I$ applies for the most preferred contract according to P_i. The contract is temporarily accepted if it is acceptable to the chosen firm.
- **Step $\boldsymbol{k}$, $\boldsymbol{k \geq 2}$.** Some student $i \in I$ without a temporarily accepted contract applies for his most preferred contract $x = (i, S, f)$ which has not been rejected yet. If x is not acceptable to f, the contract is rejected: $A^k = A^{k-1}$ and $A^k_f = A^f_{k-1}$. Otherwise, A^k and A^k_f are formed according to the principles described in Table 1.

TABLE 1. A^k and A^k_f, if the contract x is acceptable to f

<table>
<tr><td colspan="2" rowspan="2"></td><td rowspan="2">$\lvert A^{k-1}\rvert < Q$</td><td colspan="3">$\lvert A^{k-1}\rvert = Q$</td></tr>
<tr><td>$\lvert A^{k-1}_{-f_0}\rvert < R$</td><td>$\lvert A^{k-1}_{-f_0}\rvert = R$</td><td>$\lvert A^{k-1}_{-f_0}\rvert > R$</td></tr>
<tr><td rowspan="2">$f \neq f_0$</td><td>$\lvert A^{k-1}_f\rvert < Q_f$</td><td>I
$A^k_f = A^{k-1}_f \cup \{x\}$
$A^k = A^{k-1} \cup \{x\}$</td><td>II
$A^k_f = A^{k-1}_f \cup \{x\}$
$A^k = A^{k-1}_{-f_0} \cup \{x\} \cup C(A^{k-1}_{f_0})$</td><td colspan="2">III
$A^k_f = A^{k-1}_f$, if $x \notin A^k$
$A^k_f = A^{k-1}_f \cup \{x\}$ or
$C(A^{k-1}_f \cup \{x\})$, if $x \in A^k$
$A^k = C(A^{k-1} \cup \{x\})$</td></tr>
<tr><td>$\lvert A^{k-1}_f\rvert = Q_f$</td><td colspan="4">IV
$A^k_f = C(A^{k-1}_f \cup \{x\})$
$A^k = A^{k-1}_{-f} \cup A^k_f$</td></tr>
<tr><td colspan="2">$f = f_0$</td><td>V
$A^k_f = A^{k-1}_f \cup \{x\}$
$A^k = A^{k-1} \cup \{x\}$</td><td colspan="2">VI
$A^k_f = C(A^{k-1}_f \cup \{x\})$
$A^k = A^{k-1}_{-f} \cup A^k_f$</td><td>VII
$A^k_f = A^{k-1}_f$, if $x \notin A^k$
$A^k_f = A^{k-1}_f \cup \{x\}$ or
$C(A^{k-1}_f \cup \{x\})$, if $x \in A$
$A^k = C(A^{k-1} \cup \{x\})$</td></tr>
</table>

The mechanism terminates when there are no new applications. The temporary allocation becomes the final allocation μ. For each $i \in I$, $\mu(i)$ will denote the firm that i has obtained. $\mu(S)$ will denote the set of students with accepted contracts. For each firm, $\mu(f)$ will denote the set of students who have signed a contract with that firm.

Example 1. Suppose there are 5 students, 3 firms (f_0, f_1, f_2), and the school S. The school has a capacity $Q = 4$ and a reserve size $R = 2$. The capacities of f_1 and f_2 are equal to 2. Student numeration is consistent with P (i_1 is the most preferred student by the school; i_5 is the least preferred student by the school).

- **Step 1.** i_1 applies for his most preferred contract $\alpha^{i_1} = (i_1, S, f_0)$. α^{i_1} is acceptable to f_0, so $A^1 = A^1_{f_0} = \{\alpha^{i_1}\}$, while $A^1_{f_1} = A^1_{f_2} = \varnothing$.
- **Step 2.** i_2 applies for his most preferred contract $\alpha^{i_2} = (i_2, S, f_1)$. α^{i_2} is not acceptable to f_1, so still $A^2 = A^2_{f_0} = \{\alpha^{i_1}\}$, while $A^2_{f_1} = A^2_{f_2} = \varnothing$.
- **Step 3.** i_3 applies for his most preferred contract $\alpha^{i_3} = (i_3, S, f_2)$. α^{i_3} is acceptable to f_2. We should apply the conditions from

Table 2. Students' preferences

i_1	i_2	i_3	i_4	i_5
f_0	f_1	f_2	f_0	f_1
f_1	f_0	f_0	f_2	f_2
f_2	f_2	f_1	f_1	f_0

Table 3. Firms' preferences

f_0	f_1	f_2
i_1	i_1	i_1
i_2	i_3	i_2
i_3	i_5	i_3
i_4		i_4
i_5		

segment I in Table 1. Therefore, $A^3 = \{\alpha^{i_1}, \alpha^{i_3}\}$, $A^3_{f_0} = \{\alpha^{i_1}\}$, $A^3_{f_1} = \varnothing$, $A^3_{f_2} = \{\alpha^{i_3}\}$.

- **Step 4.** i_4 applies for his most preferred contract $\alpha^{i_4} = (i_4, S, f_0)$. α^{i_4} is acceptable to f_0. We should apply the conditions from segment V in Table 1. Therefore, $A^4 = \{\alpha^{i_1}, \alpha^{i_3}, \alpha^{i_4}\}$, $A^4_{f_0} = \{\alpha^{i_1}, \alpha^{i_4}\}$, $A^4_{f_1} = \varnothing$, $A^4_{f_2} = \{\alpha^{i_3}\}$.
- **Step 5.** i_5 applies for his most preferred contract $\alpha^{i_5} = (i_5, S, f_1)$. α^{i_5} is acceptable to f_1. We should apply the conditions from segment I in Table 1. Therefore, $A^5 = \{\alpha^{i_1}, \alpha^{i_3}, \alpha^{i_4}, \alpha^{i_5}\}$, $A^5_{f_0} = \{\alpha^{i_1}, \alpha^{i_4}\}$, $A^5_{f_1} = \{\alpha^{i_5}\}$, $A^5_{f_2} = \{\alpha^{i_3}\}$.
- **Step 6.** i_2 applies for his most preferred contract that has not been rejected yet $\beta^{i_2} = (i_2, S, f_0)$. β^{i_2} is acceptable to f_0. We should apply the conditions from segment VI in Table 1. Therefore, $A^6 = \{\alpha^{i_1}, \beta^{i_2}, \alpha^{i_3}, \alpha^{i_5}\}$, $A^6_{f_0} = \{\alpha^{i_1}, \beta^{i_2}\}$, $A^6_{f_1} = \{\alpha^{i_5}\}$, $A^6_{f_2} = \{\alpha^{i_3}\}$.
- **Step 7.** i_4 applies for his most preferred contract that has not been rejected yet $\beta^{i_4} = (i_4, S, f_2)$. β^{i_4} is acceptable to f_2. We should apply the conditions from segment VI in Table 1. Therefore, $A^7 = \{\alpha^{i_1}, \beta^{i_2}, \alpha^{i_3}, \beta^{i_4}\}$, $A^7_{f_0} = \{\alpha^{i_1}, \beta^{i_2}\}$, $A^7_{f_1} = \varnothing$, $A^7_{f_2} = \{\alpha^{i_3}, \beta^{i_4}\}$.
- **Step 8.** i_5 applies for his most preferred contract that has not been rejected yet $\beta^{i_5} = (i_5, S, f_2)$. β^{i_5} is not acceptable to f_2. Therefore, $A^8 = \{\alpha^{i_1}, \beta^{i_2}, \alpha^{i_3}, \beta^{i_4}\}$, $A^8_{f_0} = \{\alpha^{i_1}, \beta^{i_2}\}$, $A^8_{f_1} = \varnothing$, $A^8_{f_2} = \{\alpha^{i_3}, \beta^{i_4}\}$.
- **Step 9.** i_5 applies for his most preferred contract that has not been rejected yet $\gamma^{i_5} = (i_5, S, f_0)$. γ^{i_5} is acceptable to f_0. We should apply the conditions from segment VI in Table 1. Therefore, $A^9 = \{\alpha^{i_1}, \beta^{i_2}, \alpha^{i_3}, \beta^{i_4}\}$, $A^9_{f_0} = \{\alpha^{i_1}, \beta^{i_2}\}$, $A^9_{f_1} = \varnothing$, $A^9_{f_2} = \{\alpha^{i_3}, \beta^{i_4}\}$.

All contracts that are acceptable for the only student without a temporarily accepted contract (i_5) have already been rejected.

Hence, the mechanism finishes its work and the temporary allocation becomes the final allocation.

2.3. Definitions.

Definition 1. Firms' preferences (P_F) are responsive to P if for each $f \in F$ the relative ranking of students in P_f coincides with the relative ranking of these students in P.

Example 2. Here we provide an example of firms' preferences that are responsive and non-responsive to the school's preferences P.

Table 4. Firms' preferences are responsive to P

P	P_{f_1}	P_{f_2}
i_1	i_1	i_2
i_2	i_3	i_4
i_3		
i_4		

Table 5. Firms' preferences are not responsive to P

P	P_{f_1}	P_{f_2}
i_1	i_3	i_2
i_2	i_1	i_4
i_3		
i_4		

Definition 2. An allocation μ is stable with respect to P_I and P_F if:

(1) $\nexists i \in I : \varnothing P_i \mu(i)$
(2) $\forall f \in F \ \nexists i \in \mu(f) : \varnothing P_f i$
(3) $\nexists i \in I$ and $f \in F$ that form a blocking pair:
- either $f P_i \mu(i)$ and $i P_f j : j \in \mu(f)$;
- or $f P_i \mu(i) P_i \varnothing$ and $|\mu(f)| < Q_f$.

3. Results

Theorem 1. *If firms' preferences P_F are not responsive to P, the mechanism φ is not stable with respect to P_I and P_F.*

Proof. Suppose we have 4 students and 3 firms (f_0, f_1, f_2). The school's capacity (Q) and reserve size (R) are equal to 4 and 2 respectively. The capacity of each firm except f_0 is equal to 1. Student numeration is consistent with P (i_1 is the most preferred student by the school; i_4 is the least preferred student by the school). Other preferences are specified as follows:

We can see that P_{f_1} are not responsive to P.

TABLE 6. Students' preferences

i_1	i_2	i_3	i_4
f_0	f_2	f_1	f_1
			f_0

TABLE 7. Firms' preferences

f_0	f_1	f_2
i_1	i_4	i_2
i_2	i_3	i_4
i_3		
i_4		

The mechanism φ produces the following allocation μ:

- i_1 - f_0
- i_2 - f_2
- i_3 - f_1
- i_4 - f_0

There is a blocking pair between i_4 and f_1: i_4 prefers f_1 to $f_0 = \mu(i_4)$, while f_1 prefers i_4 to $i_3 = \mu(f_1)$. Therefore, the mechanism φ is not stable with respect to P_I and P_F. □

Suppose that we have some firm $f \in F \setminus f_0$ and at some step k this firm achieves occupancy of all places for the first time. Suppose also that the position in P of the weakest contract (according to P) temporarily accepted by f by the end of each step equals $\mathcal{P}$. Then $\mathcal{P}$ does not increase after step k.

Proof. Suppose that the weakest (according to P) contract temporarily accepted by f by the end of step k is x. The position of x in P equals $\mathcal{P}$. Further changes in the structure of contracts temporarily accepted by f are possible only in accordance with the rules described in segments III and VII of Table 1. By this time, all reserved places at the school will be filled. The changes will be a consequence of the acceptance by S of contracts that are stronger than at least x according to P. Suppose that these changes have happened. But then any contract $y : xPy$ must satisfy the conditions in segment III or IV to be temporarily accepted by f (y must fall into $C(A^{l-1} \cup \{y\})$ or into $C(A_f^{l-1} \cup \{y\})$ at some step $l > k$). It is not possible, since for any contract $z \in A^{l-1}$ the following is true: $zPxPy$. Hence, $\mathcal{P}$ cannot increase after step k. □

Theorem 2. *If firms' preferences P_F are responsive to P, the mechanism φ always produces a stable allocation.*

Proof. The first two conditions of stability hold automatically, since

(1) we assume that students never apply for unacceptable for them contracts;

(2) the mechanism ensures that firms always reject unacceptable for them contracts.

Let us show that the mechanism we propose never creates blocking pairs between students and firms. We will consider two cases.

Case 1. Suppose that there are $i, j \in I$ and $f \in F : fP_i\mu(i)$ and $iP_fj : j \in \mu(f)$. There are also contracts $x = (i, S, f)$ and $y = (j, S, f)$.

There are 5 possible paths that could lead to this outcome:

(1) f accepted y and rejected x later;
(2) f rejected x and accepted y later;
(3) f accepted x, accepted y and rejected x later;
(4) f accepted y, accepted x and rejected x later;
(5) f accepted x, then rejected x and accepted y later.

Now we will show that neither of these paths is in fact possible:

(1) The contract x could be rejected immediately only if $x \notin C(A_f^{k-1} \cup \{x\})$ or $x \notin C(A^{k-1} \cup \{x\})$, depending on the case (see Table 1). Since xP_fy and firms' preferences are responsive, xPy as well. Therefore, if $y \in A_f^{k-1}$, $x \in C(A_f^{k-1} \cup \{x\})$ and $x \in C(A^{k-1} \cup \{x\})$, as the school prefers x at least to y. Hence, if f accepted y, it could not rejected x later.

(2) The contract x could be rejected immediately only if $x \notin C(A_f^{k-1} \cup \{x\})$ or $x \notin C(A^{k-1} \cup \{x\})$, depending on the case (see Table 1). It means that each contract temporarily accepted by f was ranked higher than x by the end of step k.

First, let us suppose that $x \notin C(A^{k-1} \cup \{x\})$. It means that reserves were already filled by the time when i applied ($|A_{-f_0}^{k-1}| \geq R$). Therefore, subsequently, the school could only accept contracts ranked higher than x, since any such contract z would have to fall into either $C(A^{l-1} \cup \{z\})$ or $C(A_f^{l-1} \cup \{z\})$, where $l > k$. Thus, if $x \notin C(A^{k-1} \cup \{x\})$, the school could not accepted y later.

Second, let us suppose that $x \notin C(A_f^{k-1} \cup \{x\})$. It means that either f had no vacant places (if $f \neq f_0$) or reserves were not overfilled (if $f = f_0$).

In the first case, acceptance of y is impossible according to Lemma 2, since any contract in A_f^k is ranked higher than y by the school.

In the second case, the acceptance of y is possible either through $C(A_f^{l-1} \cup \{y\})$ (if reserves are still not overfilled by the step l) or through $C(A^{l-1} \cup \{y\})$ (if reserves are overfilled). If reserves are not overfilled, $y \notin C(A_f^{l-1} \cup \{y\})$, since the school could accept only contracts ranked higher than x. The first transition from $|A^{k-1}| = R$ to $|A^{k-1}| > R$ is possible only if the weakest student in the school holds the contract with f_0. This contract is ranked higher than x. Let us suppose that the first transition has happened. Our mechanism ensures that all contracts accepted after the transition are ranked higher than x as well. Therefore, $y \notin C(A_f^{l-1} \cup \{y\})$ or $y \notin C(A^{l-1} \cup \{y\})$, depending on the case.

Hence, if f rejected x, it could not accepted y later.

(3) Temporarily accepted contract x could be rejected if $x \notin C(A_{f_0}^{k-1})$ or $x \notin C(A_f^{k-1} \cup \{z\})$ or $x \notin C(A^{k-1} \cup \{z\})$, where z - some new submitted contract. Since temporarily accepted contracts x and y include the same firm and xPy, in all these cases y should be rejected first, not x. Therefore, f could not accepted both x and y and then rejected x.

(4) Here we can apply exactly the same logic as in the previous point. f could not accepted both x and y and then rejected x.

(5) Temporarily accepted contract x could be rejected if $x \notin C(A_{f_0}^{k-1})$ or $x \notin C(A_f^{k-1} \cup \{z\})$ or $x \notin C(A^{k-1} \cup \{z\})$, where z - some new submitted contract. It means that each contract temporarily accepted by f was ranked higher than x by the end of step k. Therefore, we can apply the same logic as in the point 2. If f rejected x, it could not accepted y later.

We have shown that **Case 1** is not possible under our mechanism. Now we will consider **Case 2**. □

4. Conclusion

References