The Method of Aggregating Expert Assessments Considering the Reliability and Caution of Experts

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The problem of aggregating information from different sources is one of the important problems of economic informatics. Various instrumental methods can be used to solve this problem. Economic information, especially expert information, has a high degree of uncertainty, inaccuracies, sources of information may have different reliability. Therefore, the mathematical tools that are supposed to be used to aggregate such information should simulate these features. In particular, the Dempster — Shafer theory of evidence (the theory of belief functions) [1,2] provides great opportunities for modeling uncertainty, inaccuracy, and reliability of information sources. In addition, the toolkit for aggregating (combining) information sources described using the so-called evidence bodies has been developed in the belief functions theory.

The report will show how the methods of the theory of confidence functions can be used to select from a set of information sources those that are least conflicting with each other considering the reliability of these sources and the caution of experts for subsequent aggregation.

Let $\mathcal{A}$ be some finite subset of nonempty sets from the set of $X \subseteq \mathbb{R}$ (set of focal elements). Some non-negative set function (mass function) m, defined on subsets of X, which satisfies the conditions: $m(A) > 0$ if $A \in \mathcal{A}$ and $\sum_{A\in\mathcal{A}} m(A) = 1$ is considered in the theory of evidence. In this case, the pair $F = (\mathcal{A}, m)$ is called the body of evidence on X.

Suppose we received information (for example, about the predicted oil price) in the form of several bodies of evidence from different sources (from different experts). We want to find a common body of evidence by combining information from several sources. Then the following questions (tasks) arise: 1) the choice of evidence bodies for combination; 2) combining several bodies of evidence into one; 3) considering the various cautions of experts and their reliability when solving the first two problems.

The choice of low-conflict evidence bodies is a general guideline for choosing evidence bodies to combining.

How to evaluate a conflict of evidence? Suppose there are two bodies of evidence $F_1 = (\mathcal{A}_1, m_1)$ and $F_2 = (\mathcal{A}_2, m_2)$. It is necessary to assess the magnitude of the conflict between the two pieces of evidence. We will use measure

$$Con^{\Gamma}(F_1, F_2) = \sum_{A\in\mathcal{A}_1, B\in\mathcal{A}_2} \gamma(A,B) m_1(A) m_2(B), \tag{1}$$

where $\Gamma = \left(\gamma(A,B)\right)_{A,B\in\mathcal{A}}$ is a matrix of values of the normalized (i. e. $0 \le \gamma(A,B) \le 1$) distance function (measure of difference) between the intervals $A, B \in \mathcal{A}$ on the $\mathbb{R}$, satisfying the conditions: 1) $\gamma(A,B) = 1$, if $A \cap B = \varnothing$; 2) $\gamma(A,A) = 0$ $\forall A \subseteq X$. Let $|A|$ be the Lebesgue measure of a set A on a line $\mathbb{R}$.

The measure of the conflict (1) will be coordinated with the combination of the bodies of evidence $F_1 = (\mathcal{A}_1, m_1)$ and $F_2 = (\mathcal{A}_2, m_2)$ according to the rule $F_S = (\mathcal{A}, m_S) = F_1 \otimes_S F_2$, where

$$m_S(C) = \frac{1}{K} \sum_{A \cap B = C} s(A,B) m_1(A) m_2(B),$$

$s(A,B) = 1 - \gamma(A,B)$, $K = \sum_{A,B} s(A,B) m_1(A) m_2(B)$.

The reliability of experts can be taken into account using Shafer's discounting rule [2]: if $\eta \in [0,1]$ characterizes the degree of reliability of the information source ($\eta = 1$ is an absolutely reliable source), then instead of the body of evidence $F = (\mathcal{A}, m)$ we will consider the body of evidence $F^{(\eta)} = (\mathcal{A}, m^{(\eta)})$, where $m^{(\eta)}(A) = \eta \cdot m(A)$ $\forall A \neq X$ and $m^{(\eta)}(X) = 1 - \eta + \eta \cdot m(X)$.

We will consider blur - a fuzzy set $\tilde{A}$ instead of the focal element $A = [a_1, a_2)$ to consider the varying degrees of caution of the expert (DM) in their assessments. Various blurring strategies are possible depending on the information about the degree of care of the decision maker:

1) if the DM estimates are careful, as a rule, then $\operatorname{supp} \tilde{A} = A$ (internal blur);
2) if the DM estimates are too accurate, as a rule, then $\ker \tilde{A} = A$ (external blur);
3) if the DM estimates are neutral, then $\mathrm{EI}[\tilde{A}] = \bar{A}$, where EI is the expected interval of a fuzzy number (neutral blur).

The report will analyze such properties of the measure of conflict (1) with respect to changes associated with discounting masses and blurring of focal elements, such as: 1) stability to small changes; 2) monotony about changes; 3) direction of change.

In addition, an example of the application of the considered conflict assessment method will be given considering the reliability and caution of experts in the choice of analysts' forecasts for the aggregation of oil prices, as well as a discussion of the qualitative results of the aggregation.

References

1. Dempster, A.P. Upper and Lower Probabilities Induced by a Multivalued Mapping// The Annals of Mathematical Statistics, 1967, 38(2), 325–339.
2. Shafer, G.: A Mathematical Theory of Evidence. Princeton University Press (1976).