Optimal Control for Market Microstructure Problems

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It is important to understand how do market participants trade to analyse stock market. Practice shows that the stock market is not absolutely elastic. Limited elasticity of supply and demand or liquidity can influence participants' behaviour significantly, which in turn can influence supply/demand or prices. In this work we consider the problem of finding an optimal strategy of executing a given order, which is also called the optimal execution problem. This problem is faced by big institutional inverstors who need to execute large orders and abort deals often in order to manage their trading expenses.

The problem of optimal execution has been addressed in several papers. Bertsimas and Lo[1] proposed a static price influence function and defined an optimal execution strategy based on minimizing the expected cost of executing a given order.

Almgren and Chriss[2] constructed a model in which the impact of a trade on price is described by a static price impact function. This function depends only on the size of the trade and does not reflect the intertemporal properties of supply/demand for a security. In addition, time is defined discretely in the model, i.e., trading time is set at certain intervals, which does not allow determining the optimal time for making trades.

Obizhaeva and Wang[3] propose a model that incorporates intertemporal properties into the market structure. When optimally timing trades, the optimal execution strategy in this model differs significantly from those proposed in previous works. This model includes a combination of discrete and continuous trades. A limit order book is used to describe the dynamics of supply and demand, with the optimal strategy typically involving an initial discrete trade that deviates the limit order book from its steady state. This deviation attracts new orders to the book. The initial trade size is chosen to attract a sufficient number of new orders at the desired prices. This is

[1]Bertsimas D., Lo A. W. Optimal control of execution costs //Journal of financial markets. 1998. Vol. 1. №1. P. 1–50.

[2]Almgren R., Chriss N. Value under liquidation //Risk. 1999. Vol. 12. №12. P. 61–63.

[3]Obizhaeva A. A., Wang J. Optimal trading strategy and supply/demand dynamics //Journal of Financial markets. 2013. Vol. 16. №1. P. 1-32.

followed by a set of continuous trades to select new orders and maintain the inflow to the book. At the end of the trading period, a final trade is executed to fill all remaining orders. This type of optimal strategy leads to interesting conclusions about the dynamics of trading volume, liquidity, and security prices. In this paper, the Obizhaeva-Wang model is extended in two directions: the drift of the fundamental part of the price and the uneven distribution of the order book are added. This leads to the problem of impulse optimal control.

In this paper, a purchase strategy is found that satisfies the necessary optimality conditions, and the dependence of this strategy on the nonlinearity coefficient is studied. In particular, it is shown that the presence of a positive drift leads to an increase in the value of the initial purchase and a decrease in subsequent purchases, and an increase in the volume of applications at a price can make a one-time purchase of the entire required volume optimal. In addition, the maximum principle is proven for the problem of impulse optimal control in a special case.

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