Predatory Trading in a Rational Market

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Abstract

I study predatory trading in a model where the predators and the prey trade with competitive, rational hedgers. Both the prey's distress and market depth are endogenous. On the one hand, limited depth helps predators move prices to push the prey into distress. On the other hand, the mere anticipation by hedgers of the prey's firesale lowers the current asset price and makes price impact trader-specific. The prey's price impact decreases before firesales, while predators' increases, making predation cheaper for them. The model predicts that predatory trading occurs in sufficiently thin markets, and shows that short-selling bans may be ineffective against predatory trading.

Keywords: Predatory trading, Firesales, Liquidity, Trader-specific Price Impact
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1 Introduction

There is anecdotal and empirical evidence that traders engage in predatory trading against financially constrained rivals to benefit from firesale prices.[1] Consider, for instance, a trader with a long position in an asset. He may have to unwind it at short notice if prices fall sufficiently, due to, e.g., a margin call or share redemptions. Predators may sell to tighten the trader's constraint and may succeed if they have enough price impact. However, predators' price impact depends on how the rest of the market reacts to predatory trading. Does the rest of the market take advantage of the predators' sales to buy at a good price, thereby bringing liqudity and cushioning the predators' impact? Or do these investors sell in anticipation of future firesale prices, thereby withdrawing liquidity?

Extant models of the literature are silent about these questions.[2] Indeed, it is common to assume that the predators and the prey trade with long-term value traders, modeled as an exogenous downward-sloping demand curve. Thus, these investors disregard short-term price movements, a less-than-fully rational behaviour. In particular, long-term value traders ignore the fact that the prey's firesale may push prices further down.

In this paper, I consider a model of predatory trading in a rational market. I replace long-term value traders by rational hedgers, who trade to share risk and determine their demand optimally given the anticipated price path. I find two main results. First, the fact that hedgers are rational does not necessarily cushion the predators' price impact: in fact, the hedgers' reaction may reduce the cost of manipulating the price for predators and contribute to induce the prey's distress. The reason is that in a rational market, the anticipation of a firesale is reflected in the price ex-ante, making it harder for the prey to meet her price-based constraint. Further, the anticipation of the prey's firesale also implies that the marginal value of trading with the prey before the firesale decreases. Indeed, why share risk with the prey if she is forced to unwind this trade in the near future? As a result, the prey's price impact decreases, while the predators' increases. It thus becomes easier for predators to move prices, and more difficult for the prey, effectively lowering the cost of

[1] For instance, there is evidence of predatory trading against LTCM in 1998 (Cai, 2009) and against several hedge funds during the 2007-2009 financial crisis, in particular in the aftermath of Bear Stearns' and Lehmann Brothers' collapse. More generally, there is evidence of predatory trading around predictable trades in equity and bond markets (Chen, Hanson, Hong, and Stein (2008), Liu (2015), Takahashi and Xu (2016), Barbon et al., (2019)).

[2] See e.g. Brunnermeier and Pedersen (2005), Attari, Mello, and Ruckes (2005), Carlin, Lobo and Viswanathan (2007), Bessembinder et al. (2016). An exception is Pritsker (2009), where all investors are rational. The prey's distress, however, is exogenous. Thus, the effects of predatory trading on market depth before the distress and their impact on the likelihood of distress cannot be studied.

predatory trading for predators. Second, I show that in a rational market predators may not have to actively trade against the prey to trigger distress. If hedgers are very risk-averse, it is enough for predators to stay on the sideline and hoard liquidity. The anticipation of the firesale by the hedgers leads to such a price movement that the prey's constraint binds, leading to a firesale. Since hedgers may simply unwind their positions in anticipation of a firesale, trading restrictions such as short-selling bans may not work against predatory trading.

I obtain these results in a three-period model with a risky asset and a risk-free asset. There are three types of investors: one prey (e.g. a financially constrained fund), at least one predator (e.g. cash-rich funds, dealers), and a unit mass of competitive hedgers. Investors trade to share risk: the prey (*she*) and the predators (*he/they*) are risk-neutral, while hedgers are risk averse and start with some endowment in the risky asset. As a result, hedgers seek to offload their inventory, thereby demanding liquidity. The predators and the prey are imperfectly competitive and thus they internalize their price impact. The key difference between the prey and the predators is that the prey faces a financial constraint.

The prey's constraint resembles realistic margin or equity constraints. The prey must liquidate her position in the risky asset if her marked-to-market wealth falls below some threshold, e.g. due to margin calls or redemptions. The prey is initially long the asset, so that her constraint binds if the asset price falls below a certain threshold. Further, the prey cannot hold more than a certain quantity of the risky asset, i.e. her ability to lever up is limited.[3]

In the absence of financial constraints, hedgers seek to share risk by selling their endowment to the predators and the prey. Risk-sharing is limited, however, due to market power, as the predators and the prey buy less than the Pareto-optimal amount to control their price impact. As a result, the asset trades at a discount relative to fundamentals. Hedgers are the marginal asset pricers in the market, so the discount is related to the distance between their desired (Pareto-optimal) holdings and their actual holdings.

The presence of the prey's constraint may induce predators to depart from this liquidity provision strategy and engage in predatory trading. Predators may buy less or even short the risky asset in order to ensure that its price is sufficiently low today, leading to the prey's liquidation tomorrow. The predator's trade-off is as follows. On the one hand, being risk neutral, predators are natural buyers of the asset. Given their price impact, predators seek to optimally spread their buys over time. Buying less or selling today conflicts with this

[3]Both the limited borrowing capacity and the marked-to-market wealth constraint may stem from agency frictions arising in the process of delegation of funds by outside investors (Shleifer and Vishny, 1997).

objective. On the other hand, predatory trading leads to the exit of the prey, reducing competition for liquidity provision in the market. Further, the prey's liquidation itself increases the demand for liquidity, which benefits predators.[4] Note that since the prey is strategic, she may avoid a binding constraint by buying the asset in a bid to support its price. This ability is limited, however, by her leverage constraint.

My first main result has two parts: (i) predatory trading may occur in equilibrium in a rational market, as long as hedgers are sufficiently risk-averse. Intuitively, hedgers' risk aversion determines the slope of their demand curve, i.e. the elasticity of their demand. Higher risk aversion leads to a less elastic demand, which yields a higher price impact: for instance, if predators sell, hedgers require a larger discount to take the other side, i.e. predators' price impact is larger. A large price impact makes it easier to move the price against the prey, hence a predatory trading equilibrium arises. (ii) More importantly, predatory trading becomes cheaper in a rational market, as hedgers' reactions ahead of the anticipated firesale impede the prey's abilty to avoid a binding constraint. First, prices (or at least hedgers' asset valuations) fall in anticipation of a firesale, a standard efficient market response, which tightens the prey's constraint. Second, price impact becomes *trader-specific*: the prey's price impact decreases, while predators' increases. Thus, if the prey buys the asset to support its price, her trades move the price *less* than opposite orders by predators. Indeed, hedgers anticipate that for each share they sell to the prey, with some probability, that share will have to be liquidated in a firesale, reducing future liquidity. Hence selling a share to the prey provides them with only partial, temporary insurance. This reduces the gains from trading with the prey. In this sense, the reactions of rational hedgers to the possibility of predatory trading can be destabilizing: the mere anticipation of the prey's distress reduces her ability to resist predatory trading.

The second main result is that the predators may not have to sell to trigger's the prey's firesale. Indeed, hedgers are more reluctant to holding the risky asset when they believe that the prey will be distressed. As a result, they are ready to sell their endowment at a lower price. If hedgers are sufficiently risk-averse, the price fall is such that predators need not sell the asset: it may be enough for them to reduce the quantity of the asset they buy, i.e. "hoard" liquidity, and let the hedgers' trading push the prey into distress. This implies that short-selling bans may be ineffective to prevent predatory trading, and that there is no direct link between selling an asset and predatory trading.

[4]Since the prey's liquidation may happen only in the last trading round, it is equivalent to exit of the market. With more trading rounds, one could imagine that the prey returns to the market at a later date, reducing somewhat the benefit of predatory trading. The key results would still hold qualitatively.

As a corollary of this result, the likelihood of predatory trading first increases and then decreases in the size of hedgers' initial position. On the one hand, the price falls more in anticipation of a firesale if hedgers have a larger position to trade. This is because hedgers' willingness to hold the asset ahead of the firesale is lower if they have a larger inventory to start with. On the other hand, hedgers' position also affects predators' outside option, which consists in providing rather than withdrawing liquidity. If hedgers have a large enough position, liquidity provision is very profitable. As a result, an increase in hedgers' selling pressure (via an increase in their initial position in the risky asset) does not necessarily generate more predatory trading.

The paper is connected to both the predatory trading/ front-running literature, and the literature on limits to arbitrage.

The literature on predatory trading relies either on exogenous liquidity (Brunnemeier and Pedersen, 2005, Attari, Mello and Ruckes, 2005, Carlin et al., 2007, Parida and Venter, 2009, Laó, 2010, Brunnermeier and Oehmke, 2014) or exogenous distress (Pritsker, 2009).[5] Instead, this paper combine endogenous liquidity and endogenous distress. Pritsker (2009) studies a rational market with endogenous liquidity, but in a setting with *exogenous* distress, i.e. in which the prey is forced to liquidate at a given time, independently of her marked-to-market wealth. Considering endogenous distress allows me to link hedgers' optimal behaviour to the probability of predatory trading. It also generates a novel state-dependent link between market liquidity and *a trader*'s funding liquidity.

Endogenous distress is also the main difference between this paper and Carlin et al. (2007) and explains why our findings differ. I find that predatory trading is likely to occur when the slope of hedgers' demand curve is steep, while Carlin et al.'s model predicts the opposite. In my setting, high price impact allows predators to move prices to induce the prey's distress. In Carlin et al. (2007), a high price impact allows the prey to retaliate against predators in a repeated interaction.

The literature on limits to arbitrage relates market liquidity to *aggregate* funding liquidity (Gromb and Vayanos, 2002, Brunnermeier and Pedersen, 2009, Vayanos and Wang, 2012). This paper generates a novel link between market and funding liquidity at the *trader's level*. Specifically, in times of high risk-aversion, a trader's price impact becomes an increasing function of her (perceived) funding liquidity, while in times of low risk-aversion, a trader's price impact is independent of her funding liquidity.

[5] In some of these papers, predators exploit the need of others to unwind their positions, e.g. by trading ahead of them. In others, predators may both exploit the selling pressure and cause it ex-ante by trading to induce the prey's financial distress, as in this paper.

The paper proceeds as follows. Section 2 presents the model. Section 3 studies the special case where hedgers have no endowment in the risky asset. This is a clean benchmark to understand the effects of the prey's financial constraint on the equilibrium. Section 4 studies the case where hedgers have positive endowments and derives additional results. Section 5 concludes. The appendix contains the proofs.

2 Model

Environment. The model has three periods: $t = 0, 1, 2$. There is a risky asset and a risk-free asset. The risky asset is in positive net supply $S \geq 0$ and pays off a dividend $\tilde{D_2}$ at $t = 2$, with $\tilde{D_2} = D + \tilde{\varepsilon_1} + \tilde{\varepsilon_2}$, $D > 0$. Let D_t denote the conditional expected dividend at time t, i.e. $D_t = \mathbb{E}_t(D_2)$. The innovations ε_1 and ε_2, revealed at $t = 1$ and $t = 2$ respectively, are independent and identically distributed normal variables with mean 0 and variance σ^2. I denote p_t the price of the risky asset. The risk-free asset is in perfectly elastic supply and offers a return r_f normalised to 0.

Investors. There are three types of investors: hedgers, predators, and a prey. When needed, I refer to the predators and the prey collectively as strategic traders. There is a unit mass of hedgers aggregated into a representative competitive investor (denoted by superscript 0) with exponential utility over final consumption, $u(C^0) = -\exp(-\alpha C^0)$ and initial endowment $X^0_{-1} \geq 0$ in the risky asset.[6] Since they have CARA preferences, their initial wealth is irrelevant for the problem, hence I assume without loss of generality that they start with cash $B^0_{-1} = 0$.

The prey (indexed by i=1, "she") and predators ($i = 2, \ldots, n$, "he","they") are risk-neutral and imperfectly competitive. Due to market power, the prey and predators internalize their price impact. At time $t = 0, 1$, hedgers set their demand for the risky asset as a function of its price. Strategic traders compete in quantities (à la Cournot) for the risky asset, taking hedgers' demand as given.

Financial constraint. The prey's constraint is twofold. First, following a low marked-to-market wealth at time 0, the prey must liquidate her risky asset holdings at time 1. Denoting B^i_t and X^i_t investor i's position in the risk-free and risky assets at time t, we can write the prey's constraint as follows:

[6] Hedgers may stand for a competitive market-making sector. Their endowment, in this case, represent market-makers' aggregate inventory, which can result from a temporary order imbalance, in the spirit of Grossman and Miller (1988). Hedgers may also stand for the demand of two groups of local traders subject to endowment shocks in segmented markets, as in Gromb and Vayanos (2002).

Assumption 1 (Marked-to-Market Wealth Constraint) *If* $B_0^1 + X_0^1 p_0 \leq \underline{V}$, *then* $X_1^1 = 0$.

The constraint says that if her marked-to-market wealth drops below the threshold $\underline{V}$ at time 0, the prey unwinds her position X_1^1 at time 1. To fix ideas, I assume that the prey starts with a *long* position in the risky asset, $X_{-1}^1 > 0$. Results do not depend on the sign of this position.

The fact that the prey's liquidiation depends on past performance opens the door to predatory trading. In practice, financiers often rely on past performance to determine funding. For instance, investors tend to redeem shares out of mutual and hedge funds following poor performance. Prime brokers and other capital providers are also likely to pull the plug after large losses. Stop-loss thresholds and other mechanisms inside banks are also likely to generate the same outcome. Agency concerns resulting from the delegation of funds from investors to strategic traders may rationalize this behaviour: Bolton and Scharfstein (1990) show that a termination threat can arise as a disciplining device in an optimal contract, even if it exposes the agent to predation risk.

In addition to the marked-to-market wealth constraint, the prey faces a leverage constraint. Her time-0 position in the risky asset, X_0^1, is bounded above by $\bar{X}$.

Assumption 2 (Leverage Constraint) $X_0^1 \leq \bar{X}$, *where* $\bar{X} \geq X_{-1}^1$.

This assumption implies that the prey can hold more than her initial endowment at time 0 but cannot increase her position indefinitely. I denote $a = \frac{\bar{X}}{X_{-1}^1}$ the prey's leverage capacity at time 0 (i.e. $a \geq 1$). Note that x_t^i denotes the time-t trade in the risky asset of trader i, while X_t^i denotes the time t position in the risky asset after trading at time t. Thus, capital letters denote positions, while small letters denote trades, and $X_t^i = X_{t-1}^i + x_t^i$.

For simplicity, I assume that predators are cash-rich or able to secure better funding conditions and do not face any financial constraints.[7]

Information. Information is complete. Thus, investors know each others' positions, trading needs, and constraints. Knowledge of other traders' positions or impending trades may be revealed through institutional features (see, e.g. Friederich and Payne, 2014), mandatory disclosures, information leakage by brokers (Barbon, Di Maggio, Franzoni, Landier, 2019), or

[7]Strategic traders such as hedge funds may have some leeway in chosing their capital structure. For instance, some hedge funds are able to impose better lock-up periods or gates to their investors than their rivalsand is optimal differentiation in strategic traders' capital structure can arise in equilibrium in an optimal contract setting (Hombert and Thesmar (2009)).

even the financial press (e.g. the "London whale").[8] Further, it is well-known under market participants that events such as index reconstitutions, ratings downgrades, futures rolls lead to mechanical portfolio rebalancings by passive or institutional investors.

Maximization problems. Given that all market participants are informed about the prey's constraints, they take into account the possibility of her being distressed in their maximization problems. Hedgers choose positions X_0^0 and X_1^0 to maximize expected utility subject to their dynamic budget constraint, while taking prices and the prey's constraints as given. Their problem is given by:

$$
\begin{aligned}
&\max_{X_0^0, X_1^0} -\mathbb{E}_0\left[\exp\left(-\alpha C_2^0\right)\right] \\
&\quad s.t.\ W_t^0 = W_{t-1}^0 + X_{t-1}^0(p_t - p_{t-1}) \\
&\qquad\quad \text{Assumptions 1 \& 2}
\end{aligned}
$$

Given the CARA-Gaussian framework, hedgers' demand is given by

$$
X_t^0 = \frac{\mathbb{E}_t(p_{t+1}) - p_t}{\beta}, \quad \text{where } \beta = \alpha\sigma^2 \tag{2.1}
$$

The predators and the prey do not take prices as given. They choose trades x_t^i to maximize expected wealth subject to the price schedule and the prey's financial constraints. The price schedules $p_t(\sum_{j=2}^n x_t^j, x_t^1) : \mathbb{R} \to \mathbb{R}$ map the predators' and the prey's trades of time t into the equilibrium price. They are obtained by inverting hedgers' demand (2.1) and imposing market-clearing. The optimization problem of the predators and the prey is given by

$$
\begin{aligned}
&\max_{x_0^i, x_1^i} \mathbb{E}_0\left[W_2^i\right] \\
&\quad s.t.\ W_2^i = B_{-1}^i - x_0^i p_0\left(\sum_{j=2}^n x_0^j, x_0^1\right) - x_1^i p_1\left(\sum_{j=2}^n x_1^j, x_1^1\right) + X_1^i D_2 \\
&\qquad\quad \text{Assumptions 1 \& 2}
\end{aligned}
$$

[8] In 2011, the front page of the Wall Street Journal claimed "London whale rattles debt markets". Following the news, the CIO at JPMorgan was quickly identified as the large trader disrupting the CDS market and had to unwind a large position. Smaller hedge funds were allegedly trading ahead of the London whale, increasing liquidation costs to about \$6bn for JP Morgan. The collapse of the hedge fund Amaranth disrupted the natural gas market in 2007. Traders inferred Amaranth's positions by observing from the exchange data that a single market participant had accumulated very large positions in the futures market (Levin and Coleman, 2007). See also Foucault et al. (2003) and references therein for a description of non-anonymous trading environments.

We can now state the equilibrium definition.

Definition 1 (Equilibrium) *An equilibrium consists of trades x_t^i and prices p_t such that (i) Hedgers' holdings are optimal given rationally anticipated prices; (ii) given other predators' trades, the prey's trades, the prey's constraints, and the price schedules, predator i's trades maximize his expected wealth; (iii) given the predators' trades, her constraints, and the price schedules, the prey's trades maximize her expected wealth.*

Distress threshold. Since she is initially long the asset, the prey falls into distress when the price of the asset is low at time 0. Rearranging the terms in the marked-to-market wealth constraint, one can see that the prey is in distress when the price falls below the treshold $\bar{p}_0$, given by

$$\bar{p}_0 \equiv \frac{\underline{V} - B_{-1}^1}{X_{-1}^1} \tag{2.2}$$

The threshold is increasing in $\underline{V}$, which measures the severity of the constraint, and decreasing in the amount of initial cash of the prey, B_{-1}^1. I assume that parameters are such that $0 < \bar{p}_0 < D$, i.e.

Assumption 3 (Boundaries of Distress Threshold) $0 < \underline{V} - B_{-1}^1 < X_{-1}^1 D$

This assumption implies that the prey remains solvent if the asset trades at its expected value.

3 Predatory trading vs no trading

In this section, I show the first main result of the paper in the benchmark case where hedgers hold no initial position in the risky asset, i.e., the predators and the prey initially hold the entire asset supply. In this case, there are no gains from trade between hedgers and the predators and prey. Thus, absent financial constraints, there is no trading. The presence of the financial constraint, however, generates predatory trading if hedgers have a low risk-bearing capacity. In this predatory trading equilibrium, the traders' financial strength (or at least hedgers' perception of it) affects their price impact. In particular, I show that the prey's price impact decreases, while the predators' increases, which reduces the probability of survival of the prey.

3.1 Trading at time 1

The following lemma summarizes the equilibrium at time 1, depending on whether the prey is distressed or not.

Lemma 1 *When the prey is solvent, there is a unique symmetric equilibrium at time* 1*, given by:*

$$\forall i = 1, ..., n, \;\; x_1^i = \frac{-\sum_{j=1}^{n} x_0^j}{n+1} \tag{3.1}$$

When the prey is distressed, the unique equilibrium at time 1 *is given by:*

$$\begin{aligned} x_1^1 &= -X_0^1 \\ \forall i = 2, ..., n, \;\; x_1^i &= \frac{X_0^1 - \sum_{j=1}^{n} x_0^j}{n} \end{aligned} \tag{3.2}$$

Associated payoffs at time 1 can be written as $B_0^i + X_0^i D_1 + \pi_1^{i,k}$*,* $k \in \{d, nd\}$*, where*

$$\pi_1^{i,nd}\left(\sum_{j=2}^{n} x_0^j, x_0^1\right) = \beta \frac{\left(-\sum_{j\neq i} x_0^j - x_0^i\right)^2}{(n+1)^2} \qquad \text{if } p_0 > \bar{p}_0 \tag{3.3}$$

$$\pi_1^{i,d}\left(\sum_{j=2}^{n} x_0^j, x_0^1\right) = \beta \frac{\left(X_{-1}^1 - \sum_{j=2, j\neq i}^{n} x_0^j - x_0^i\right)^2}{n^2} \qquad \text{if } p_0 \leq \bar{p}_0 \text{ and } i = 2, \ldots, n \tag{3.4}$$

$$\pi_1^{i,d}\left(\sum_{j=2}^{n} x_0^j, x_0^1\right) = -\beta X_0^1 \frac{X_{-1}^1 - \sum_{j=2}^{n} x_0^j}{n} \qquad \text{if } p_0 \leq \bar{p}_0 \text{ and } i = 1 \tag{3.5}$$

When the prey is not distressed, the predators and the prey trade the same quantity (3.1). They revert a fraction of the time-0 aggregate trade to smooth price impact. When the prey is distressed, she liquidates her holdings of the risky asset, thus behaving as a liquidity trader in need of immediacy. The predators each buy a fraction $\frac{1}{n}$ of the prey's holdings, thus not fully absorbing her order (since $\frac{n-1}{n} < 1$), which implies that hedgers will have to absorb the residual order. As before, traders smooth their price impact by going against the aggregate order flow. Note that the denominator in equation (3.2) decreases, reflecting the higher degree of predators' market power. This is because the prey no longer acts strategically, so that only $n-1$ predators participate in the Cournot game at time 1.

3.2 Price schedule at time 0

Because there is still uncertainty at time 1 about the fundamental value of the asset, hedgers are unwilling to hold large quantities at time 0. Since hedgers understand that predators gain further market power during firesales, their demand depends on whether they expect a firesale or not at time 1. This affects the properties of the price schedule faced by the predators and the prey at time 0.

Lemma 2 *Let p_0^{nd} and p_0^d denote the price schedule when hedgers expect no-distress and distress, respectively. The price schedule depends on the hedgers' beliefs about future distress as follows:*

$$p_0^{nd}\left(\sum_{j=2}^{n} x_0^j, x_0^1\right) = D + \beta\frac{n+2}{n+1}\sum_{j=1}^{n} x_0^j \tag{3.6}$$

$$p_0^{d}\left(\sum_{j=2}^{n} x_0^j, x_0^1\right) = D - \beta\frac{1}{n}X_{-1}^1 + \beta\frac{n+1}{n}\sum_{j=2}^{n} x_0^j + \beta x_0^1 \tag{3.7}$$

Comparing equations (3.6) and (3.7) shows that two effects take place when hedgers anticipate distress: first the intercept of the price schedule decreases; second, price impact becomes *trader-specific*. In particular, the prey's trades now move the price less than predators', while all traders have the same price impact when hedgers expect no distress.

The intuition for the first effect is that hedgers' valuation for the risky asset decreases when they believe that the prey will be distressed in the next period. Since the prey's liquidation will push the price down at time 1, hedgers would be ready to, say, sell at a lower price at time 0.

The second effect arises because the price impact coefficients reflect the differential marginal gains from trading with each type of trader. If hedgers anticipate distress, their maginal gain from selling to the prey is lower than to predators at time 0. Predators will keep this asset until time 2, i.e., until the asset pays off and returns to perfect liquidity. This is not the case when selling to the prey: if hedgers' anticipations are correct, as they are in equilibrium, the asset sold at time 0 to the prey will return to the market at time 1, when predators gain market power.

Trader-specific price impact has been documented at least since Chan and Lakonishok (1995). However, to the best of my knowledge there is no empirical test linking an individual trader's price impact to his funding liquidity or past performance. Cai (2009) finds that LTCM's price impact was on average lower in the months before receiving margin calls in September 1998 than during the crisis itself.

3.3 Equilibria at time 0

The fact that the price schedule depends on hedgers' expectations about the prey's distress leads to multiple equilibria at time 0. Taking hedgers' beliefs as given, I determine conditions under which no trading and predatory trading arise in equilibrium.

3.3.1 No trading

Suppose that hedgers anticipate no trading, and thus no distress given Assumption 3.[9] Given that hedgers have no endowment, there are no gains from trade between them and the predators and the prey. However, the possibility of the prey's distress means that there may be gains from trading against the prey for predators. I determine under which conditions a no trading equilibrium arises.

First, I verify that it is never in the interest of the prey to voluntarily exit the market (proof of Proposition 4 in the appendix). One could imagine that the prey could make positive trading gains by engaging in some price manipulation, e.g., buy at a discount at time 0 and sell to the predators at time 1. Equation (3.6) shows that any purchase at time 0 pushes the price above the expected dividend, so that the prey would have to sell at time 0 to push the price under the distress threshold. Thus, the prey would sell progressively her endowment. However, the prey's price impact at time 0 implies that this strategy yields negative trading gains, so the prey is better off not trading.

The predators, however, may have an incentive to deviate from the no-trading situation to push the prey into distress. This is costly, because it requires to manipulate the price and tighten the prey's financial constraint. But a deviating predator may benefit from the increase in the asset supply resulting from the prey's firesale, and the decrease in competition among the remaining strategic traders.

Predators' trade-off. Since all predators have price impact, each of them recognizes he is pivotal for the outcome of the game. A predator faces a trade-off between manipulating the price and gaining from the prey's firesale, or not trading, which entails both no cost and

[9] From equation (3.6), if all strategic traders do not trade (i.e. submit orders $x_0^i = 0$), the asset will trade at the fundamental value - and therefore the prey will not be distressed.

no trading profit. Suppose that strategic traders (except predator i) trade $x_0^{-i} = 0$. Then predator i's problem may be written as follows:

$$\max_{x_0^i} E_{-1}^i + x_0^i \left(D - p_0^{nd}\left(x_0^i, 0\right)\right) + \pi_1^{i,nd}\left(x_0^i, 0\right) I_{p_0 > \bar{p}_0} + \pi_1^{i,d}\left(x_0^i, 0\right) I_{p_0 \leq \bar{p}_0},$$

where $E_{-1}^i = B_{-1}^i + X_{-1}^i D$ denotes the expected value of the predator's endowment and I_c a dummy variable that equals one when condition c is satisfied. Suppose that other strategic traders stick to no trading, i.e. $x_0^{-i} = 0$. If the predator chooses $x_0^i = 0$ as well, the price will be above the prey's distress threshold $\bar{p}_0$, and the predator's profit is thus $\pi_1^{i,nd} = 0$.[10] If the predators chooses to push the prey into distress, he must sell, which involves a quadratic cost: the second term in the objective function boils down to $-\frac{n+2}{n+1}(x_0^i)^2$. However, if the prey is distressed, the predator can benefit at time 1 from the decreased competition and the prey's firesale, earning $\pi_1^{i,d} > \pi_1^{i,nd}$.

Ruling out "self-fulfilling" distress. By inspecting the maximization problem, one can also see that the prey's distress may be "self-fulfilling". Namely, ex-ante, it is optimal for the predator to take a short position in the asset if he expects the prey to be distressed. Indeed, if the predator anticipates the prey's distress, he expects an increase in the asset supply and less competition in the future. Therefore the marginal cost of buying one more unit at time 1 decreases. Hence it is optimal for the predator to buy less at time 0 (i.e., here, to short the asset) and exploit the negative price pressure exerted by the prey's firesale at time 1. Since the predators' trades affect prices, the mere anticipation by a predator that the prey will be distressed at time 1 may indeed lead to a price below $\bar{p}_0$ and trigger the prey's distress. The self-fulfilling distress can be defined more formally as follows:

Definition 2 *Suppose that strategic traders $-i$ choose $x_0^{-i} = 0$. The prey's distress is self-fulfilling if $p_0^{nd}(\hat{x}_0^i, 0) \leq \bar{p}_0$, where $\hat{x}_0^i = \arg\max_{x_0^i} x_0^i \left(D - p_0^{nd}(x_0^i, 0)\right) + \pi_1^{i,d}(x_0^i, 0)$.*

To focus on predatory trading as a strategy aiming at eliminating a rival trader, I rule out self-fulfilling distress by imposing the following condition throughout:

Lemma 3 *Suppose that predators (except i) and the prey do not trade. There is no self-fulfilling distress if and only if $\beta < \bar{\beta}_{nd}$, where $\bar{\beta}_{nd}$ is given by equation (B.11). On this parameter interval, inducing distress requires predator i to trade*

$$x_0^i = \frac{n+1}{n+2}\frac{\bar{p}_0 - D}{\beta} < 0 \tag{3.8}$$

[10] Note that since $\forall n \geq 2$, $\frac{n+2}{n+1} > \frac{1}{(n+1)^2}$, all other strategies leading to $p_0 > \bar{p}_0$ are dominated by $x_0^i = 0$.

To rule out self-fulfilling distress, one must focus on situations in which hedgers' demand curve has a flat enough slope, i.e. if $\beta < \bar{\beta}_{nd}$. Intuitively, in this case, the price is not responsive enough to trades, such that a short position taken by a trader anticipating distress does not automatically lead to the prey's firesale. The predator's order, given by equation (3.8) is just enough to push the price to $\bar{p}_0$.

Proposition 1 (No Trading Equilibrium) *There exists a no-trading equilibrium in which the prey remains solvent if and only if $\beta < \underline{\beta}_{nd}$, with $0 < \underline{\beta}_{nd} < \bar{\beta}_{nd}$. Equilibrium prices are:*

$$p_0 = D \tag{3.9}$$

$$p_1 = D_1 \tag{3.10}$$

This result shows that the no-trading equilibrium holds in the presence of financial constraints only if the slope of hedgers' demand curve is flat enough. The intuition is the same as for Lemma 3 on self-fulfilling distress. If the slope is steep, a predator can easily move the price against the prey and this reduces the cost of predation. Further, a steep slope means that hedgers are reluctant to bear risk (or equivalently that the asset is very risky), implying that the firesale exerts a strong negative pressure on the price at time 1, which increases the benefit of predation.

3.3.2 Predatory trading

I now assume that hedgers believe at time 0 that the prey will be distressed in the future, so that the price schedule is given by (3.7). I conjecture that there exists an equilibrium with predatory trading such that (i) the predators push the price to the distress threshold $\bar{p}_0$ and (ii) the prey's leverage constraint is binding so it is too costly for her to stay in the market (i.e., keep the price above $\bar{p}_0$). I assume that predators trade symmetrically. This implies that the strategies are

$$x_0^1 = x_0^l \equiv \bar{X} - X_{-1}^1 \tag{3.11}$$

$$\forall\, i = 2, ..., n,\ x_0^i = x_0^p \tag{3.12}$$

where x_0^p is defined as $p_0^d((n-1)x_0^p, x_0^l) = \bar{p}_0$.

The prey's problem. The candidate equilibrium is such that it is too costly for the prey to outbid the predators, as her leverage constraint binds. Thus, the prey's problem is to maximize the proceeds of liquidating her holdings conditional on distress. Taking predators' strategy as given, the prey's problem is:

$$\max_{x_0^1 \leq \bar{X} - X_{-1}^1} E_{-1}^1 + x_0^1 \left[D - p_0^d\left((n-1)x_0^p, x_0^1\right)\right] + \pi_1^{1,d}\left((n-1)x_0^p, x_0^1\right)$$

The next result gives conditions under which it is inded optimal for the prey to be fully leveraged given predators' strategies:

Lemma 4 (Prey's Optimal Liquidation Strategy) *If $\beta < \beta_F$, the prey's best response to predators' conjectured strategy is $x_0^1 = \bar{X} - X_{-1}^1$, where β_F is given by equation (C.5).*

Intutively, if β is too large, the prey may buy less than her leverage capacity allows, leading to a price below $\bar{p}_0$. The unused leverage capacity may then be used to outbid predators and remain in the market.

Predators' problem. I now study predators' incentives to engage in predatory trading. First, it is necessary to determine the conditions under which the prey's distress is not self-fulfilling. Since the price schedule is different in this equilibrium, conditions are slightly different from the previous case:

Definition 3 *Suppose that that predators $-i$ and the prey trade the conjectured strategies (3.12) and (3.11). The prey's distress is self-fulfilling if $p_0^d\left((n-2)x_0^p + \hat{x}_0^i + x_0^l\right) \leq \bar{p}_0$, where $\hat{x}_0^i = \arg\max_{x_0^i} x_0^i\left[D - p_0^d\left((n-2)x_0^p + x_0^i, x_0^l\right)\right] + \pi_1^{i,d}\left((n-2)x_0^p + x_0^i, x_0^l\right)$.*

The conditions ruling out self-fulfilling distress involve the prey's leverage capacity, a.

Lemma 5 *There exists cutoffs $\bar{a}_n$, where $\forall n \geq 2$, $\bar{a}_n > 1$, and $\bar{\beta}_d$, given by equations (C.7) (C.8), such that*

- *If $a > \bar{a}_n$, distress is never self-fulfilling.*
- *If $a \leq \bar{a}_n$, distress is not self-fulfilling if and only if $\beta < \bar{\beta}_d$.*

The lemma shows that the prey's distress can not stem from a self-fulfilling predatory trading strategy if her leverage capacity is large enough. If the prey has enough dry powder, she does not "automatically" fall into distress, because her trades support the price sufficiently in the conjectured equilibrium. If the prey has little dry powder, i.e. a low, the prey's distress is not self-fulfilling as long as hedgers' demand curve is not too steep, i.e. if the price is not too responsive to trades.

So far, Lemma 5 and 4 imply that the relevant parameter space for these strategies is $\beta \in \left]0, \bar{\beta}_d \wedge \beta_F\right[$. I show in the appendix that in the special case where $X^0_{-1} = 0$, $\beta_F < \bar{\beta}_d$, so that the relevant interval is $\beta \in]0, \beta_F[$.

Predator i engages in predatory trading if the payoff from doing so, $J_0^{i,p}$ is higher than that of rescuing the prey, $J_0^{i,r}$, where

$$
\begin{aligned}
J_0^{i,p} =& E^i_{-1} + x_0^p[D - p_0^d((n-1)x_0^p, x_0^l)] + \pi_1^d((n-1)x_0^p, x_0^l) && (3.13)\\
J_0^{i,r} =& \max_{x_0^i} E^i_{-1} + x_0^i[D - p_0^d((n-2)x_0^p + x_0^i, x_0^l)] + \pi_1^{nd}((n-2)x_0^p + x_0^i, x_0^l) && (3.14)\\
& \quad \text{s.t.} \quad p_0 > \bar{p}_0 && (3.15)
\end{aligned}
$$

Note that as distress may be self-fulfilling, so may be the prey's survival.

Definition 4 *The prey's survival is self-fulfilling if $p_0^d((n-2)x_0^p + \hat{x}_0^i, x_0^l) > \bar{p}_0$, where $\hat{x}_0^i = \arg\max_{x_0^i} E^i_{-1} + x_0^i[D - p_0^d((n-2)x_0^p + x_0^i, x_0^l)] + \pi_1^{nd}((n-2)x_0^p + x_0^i, x_0^l)$.*

Then for each case, I determine whether predator i has an incentive to rescue the prey. The reason why this may happen is that the predatory trade x_0^p does not necessarily solve the optimal trading split between time 0 and time 1. Since distress is not self-fulfilling, predators must depart from their optimal trade, which is costly. If this cost is larger than the benefit of reduced competition and higher supply at time 1, a predator may deviate and rescue the prey.

Equilibrium. The predators' trade-off yields an additional necessary condition on β.

Proposition 2 (Predatory Trading Equilibrium) *There exists a predatory trading equilibrium characterized by equations (3.11)-(3.12) iff $\beta \in \left[\underline{\beta}_d \wedge \beta_F, \beta_F\right[$, with $\underline{\beta}_d > 0$.*

The intuition for this result mirros that of the no trading equilibrium. If price impact is large enugh (high β), inducing the prey's distress is not too costly, hence predators engage in predatory trading. Further, in this case, the prey's firesale is likely to exert strongly negative price pressure, since the hedgers have a limited risk-bearing capacity.

Corollary 1 *The equilibrium threshold $\underline{\beta}_d$ is lower when the prey is more exposed to the risk of forced liquidation (high $\underline{V}$) or has less cash (low B^1_{-1}).*

If the prey is more constrained, the cost of the predatory trading strategy is lower, hence the condition on β is less strict.

3.3.3 Coexistence of no-trading and predatory trading equilibria

From Proposition 1 and 2, I get:

Proposition 3 *When $X_{-1}^0 = 0$,*

- *The no-trading equilibrium is the only equilibrium for* $\beta \in \left]0, \min\left(\beta_F, \underline{\beta}_d, \underline{\beta}_{nd}\right)\right]$.
- *It coexists with the predatory trading equilibrium on* $\left]\min\left(\beta_F, \underline{\beta}_d, \underline{\beta}_{nd}\right), \min\left(\beta_F, \bar{\beta}_d, \underline{\beta}_{nd}\right)\right[$.
- *Predatory trading is the only equilibrium on* $\left[\min\left(\beta_F, \bar{\beta}_d, \underline{\beta}_{nd}\right), \beta_F\right[$.

To understand further in which circumstances equilibria may coexist and when predatory trading is the only equilibrium, I consider:

$$\bar{\beta}_d - \beta_F = \frac{D - \bar{p}_0}{\bar{X}} f(n, a)$$

where the function f is given by equation (D.1) in the appendix. The predatory trading equilibrium is the only equilibrium on a non-empty interval if $f(n, a) > 0$. Since f is monotonically increasing in a, the function implicitly defines a cutoff $a^*(n)$ such that:

$$f(n, a^*(n)) = 0$$

Hence the predatory trading equilibrium exists if $a \leq a^*(n)$. Panel (a) of Figure 1 plots the cutoff a^* (red dotted line), and shows that the predatory trading equilibrium exists when both the number of predators and the prey's leverage capacity are small. Intuitively, if there are many predators, fierce competition during the prey's firesale will quickly erode the benefit of predatory trading - and more quickly than it decreases the cost per predator. Hence coordination on the predatory trading equilibria is more difficult to obtain. When the prey has a high leverage capacity, the cost of inducing distress is high, hence predatory trading is less likely.

Panel (a) of Figure 1 also features a second cutoff $\hat{a}^*(n)$ defined as

$$g(n, \hat{a}^*(n)) = 0, \text{where } \underline{\beta}_{nd} - \underline{\beta}_d = \tfrac{D - \bar{p}_0}{\bar{X}} g(n, a)$$

Since g is monotonically decreasing in a, the no-trading and predatory trading equilibria coexist (that is, $\underline{\beta}_{nd} > \bar{\beta}_d$) when $a \geq \hat{a}^*(n)$, i.e. in the region above the full dark blue line. Hence, it is only when the prey is very constrained in terms of leverage, and the group of predators very concentrated that predatory trading is the only equilibrium. The model therefore delivers a clear prediction in this case, in spite of the self-fulfilling nature of the equilibria.

In the region defined by $a \leq \hat{a}^*(n)$, the model produces the "net" probability of predatory trading (i.e. excluding the region where both equilibria coexist).

Corollary 2 *Suppose that $a \leq \hat{a}^*(n)$ and denote $q(n,a) = 1 - \frac{\beta_{nd}}{\beta_F}$ the net probability of predatory trading. Then q decreases linearly in a, the prey's leverage capacity.*

It is costly to engage in predatory trading against the prey if she has a lot of dry powder. Hence the probability of predatory trading q decreases in a. To understand the effect of the number of predators, I plot q in Panel (b) of Figure 1. The graph shows that the probability decreases with n, the number of predators, and decreases faster when n is small, a non-linear effect. This is because the benefit of predatory trading decreases as $\frac{1}{n^2}$.

3.4 Implications

3.4.1 Cost of predatory trading

Pushing the asset price to the prey's liquidation threshold $\bar{p}_0$ is costly for predators. Absent financial constraint, all predators would trade zero. With financial constraints, in the predatory equilibrium predators short the asset, although there are no risk-sharing motives to do so. Thus, I define the cost of predation as the distance between the predators' aggregate trade $Q = \sum_{i=2}^{n} x_0^i$ in the predatory equilibrium and zero. To understand how the change in price schedule affects the cost of predatory trading, it is interesting to compare the cost that prevails when hedgers (correctly) anticipate distress, and the cost that predators would have to bear if hedgers incorrectly believed that the prey will not liquidate. Thus I compare the cost under the on-equilibrium price schedule and the off-equilibrium price schedule. To calculate Q, I fix the prey's strategy and assume that she is fully leveraged.

Lemma 6 *Suppose $X_0^1 = \bar{X}$, and let Q^d denote the cost of trading when hedgers anticipate distress and Q^{nd} when they do not. For all parameter values, predators must short less when hedgers anticipate distress, $Q^d \geq Q^{nd}$, with $Q^{nd} < 0$.*

This result shows that it becomes cheaper for predators to push the prey into distress when hedgers anticipate that the prey will eventually be forced to liquidate her positions. Each unit bought by the prey pushes up the price less than an opposite order by a predator.

Another interesting implication of the change in liquidity is that the size of the prey's initial position has an ambiguous effect on predators' time 0 trade, i.e. on the cost of predatory trading:

Corollary 3 *Denote $\bar{X} = aX_{-1}^1$, with $a \geq 1$. Then from equation (3.12), the effect of a change in the prey's initial size on predators' aggregate order Q^d is:*

$$\frac{\partial Q^d}{\partial X_{-1}^1} = \underbrace{1}_{price\ effect} + \underbrace{\frac{n}{n+1}}_{\textit{diff. price impact "multiplier"} < 1} \underbrace{\left[-a + \frac{1}{\beta}\frac{\partial R}{\partial X_{-1}^1}\right]}_{collateral\ effect\ <0}$$

where $R = \bar{p}_0 - D$.

Corollary 3 describes the impact of a small change in the prey's position on the amount predators must trade to push her into distress. Holding a larger position in the risky asset may either decrease or increase the cost of predatory trading. Remember that Q is negative. Thus, if a larger position has a positive impact, it brings Q closer to zero and reduces the cost of predatory trading. On the one hand, a larger position leads to a lower price ex-ante due to the anticipation of the firesale, which helps predators trigger the prey's distress. On the other hand, a larger position means that the prey is richer and that her distress threshold is lower (see equation (2.2)). This collateral effect makes predatory trading more costly. Interestingly, the price effect is 1, while the collateral effect is multiplied by $\frac{n}{n+1} < 1$. This is a consequence of the decrease in price impact the prey experiences in this regime. Hence the decrease in price impact reduces the benefit of holding a large position.

3.4.2 Implications for liquidity measures

Our analysis has interesting implications for liquidity measures and liquidity proxies. First, from the example above, it is clear that turnover cannot be used as a proxy for liquidity.

In the absence of the prey's financial constraints, it is optimal not to trade since the more risk-tolerant investors (the prey and the predators) initially hold the entire asset supply. In that sense, the mere presence of the financial constraint generates "excessive" trading volume. This mechanism complements existing mechanisms leadig to excess trading in the literature, such as heterogeneous information (e.g. Karpoff, 1986) or career concerns (Dasgupta and Prat, 2006). Interestingly, it is precisely when risk-aversion is high, that is when hedgers are the most unwilling to hold the asset, that they end up with some in their hands.

As shown in Lemma 2, predators' price impact increases and the prey's decreases in the predatory trading equilibrium relative to the no-trading equilibrium. Further, the aggregate price impact decreases in the sense that if all traders submit the same order, it pushes up the price less when the hedgers expect a firesale than when they do not. In spite of this, it is difficult to argue that the market is more liquid. In our context, trading volume and market depth can thus be misleading indicators of market liquidity. The only consistent measure here is the deviation of the transaction price from the risk-neutral value of the asset $\mathbb{E}_t\left(D_2\right)$.

4 Predatory trading vs liquidity provision

I now move on to the case where hedgers start with a long position in the risky asset, i.e. $X_{-1}^0 > 0$, which introduces a risk-sharing motive between strategic traders and hedgers. Thus the no-trading equilibrium is replaced by an equilibrium with imperfect liquidity provision but no distress.

This extension yields two new interesting results: (i) an increase in hedgers' endowment has an ambiguous impact on the probability of predatory trading. The ex-ante price effect increases with hedgers' endowment, decreasing the cost of pushing of predatory trading. However, an increase in hedgers' endowment increases also the benefit of providing liquidity to hedgers. (ii) Predators may no longer have to sell to induce the prey's distress: it may be enough for them to hoard liquidity, as hedgers' reluctance to holding the asset ahead of a firesale may sufficiently decrease the price to trigger distress.

4.1 Equilibria

4.1.1 Liquidity provision

I conjecture that there exists an equilibrium in which the predators and the prey buy the asset from hedgers, thereby providing them with liquidity (that is allowing them to swap the risky, illiquid asset for the safe, liquid asset). With hedgers' endowments, the price schedule based on no distress at time 1 becomes (by a slight abuse of notation, I keep the same notation for the price schedule):

$$p_0^{nd}\left(\sum_{j=2}^{n} x_0^j, x_0^1\right) = D - \beta\frac{n+2}{n+1}X_{-1}^0 + \beta\frac{n+2}{n+1}\sum_{j=1}^{n} x_0^j \tag{4.1}$$

Proposition 4 (Liquidity Provision Equilibrium) *Suppose* $0 < \beta < \bar{\beta}_{nd}$*. On this interval, there exists a unique (symmetric) no-distress equilibrium iff* $\beta < \underline{\beta}_{nd} \wedge \bar{\beta}_{nd}$ *and* $c_{0,n}X_{-1}^0 \leq \bar{X} - X_{-1}^1$.

The equilibrium trades are

$$\forall i = 1, ..., n, \; x_0^i = c_{0,n} X_{-1}^0 \quad (4.2)$$
$$x_1^i = c_{1,n} X_{-1}^0 \quad (4.3)$$

Equilibrium prices are:

$$p_0 = D - \beta \rho_{0,n} X_{-1}^0 > \bar{p}_0 \quad (4.4)$$
$$p_1 = D + \varepsilon_1 - \beta \rho_{1,n} X_{-1}^0 \quad (4.5)$$

with, $\forall n \geq 1$, $c_{0,n} > c_{1,n}$, $\rho_{0,n} > \rho_{1,n}$, $n(c_{0,n} + c_{1,n}) < 1$.

The coefficients $c_{0,n}$, $c_{1,n}$, $\rho_{0,n}$ *and* $\rho_{1,n}$ *are given by equations* (B.1)-(B.4)*, and the thresholds* $\underline{\beta}_{nd}$ *and* $\bar{\beta}_{nd}$ *by equations* (B.26) *and* (B.11) *in the appendix.*

The equilibrium conditions on β given in Proposition 4 are similar to those of Proposition 1, except that the thresholds $\underline{\beta}_{nd}$ and $\bar{\beta}_{nd}$ are now evaluated for $X_{-1}^0 > 0$.[11] The condition $c_{0,n} X_{-1}^0 \leq \bar{X} - X_{-1}^1$ ensures that the equilibrium strategy is feasible for the prey, in spite of her leverage constraint.

In equilibrium, the predators and the prey provide limited liquidity in the market. In total, they buy an amount $n(c_{0,n} + c_{1,n}) X_{-1}^0$, which is lower than the hedgers endowment ($\forall n \geq 2$, $n(c_{0,n} + c_{1,n}) < 1$). This is because the liquidity supply side of the market is oligopolistic. Yet, the equilibrium conditions ensure that the prey is not distressed: the equilibrium price is above $\bar{p}_0$. The predators and the prey spread their trades over both periods. Since trades move prices in a permanent manner, a strategic trader lowers his average purchase price by splitting up trades.

The risky asset trades at a discount because of imperfect competition. This discount decreases over time because of the gradual purchases of the strategic traders, and varies as follows:

Corollary 4 *The illiquidity discount in period* t *is* $\Gamma_t = E_t(D_2) - p_t = \beta \rho_{t,n} X_{-1}^0 > 0$ $(t = 0, 1)$.

- *At each period, the discount is larger for a higher risk-aversion coefficient* α*, a higher riskiness of the asset* σ^2*, a larger hedging need* X_{-1}^0*, a smaller number* n *of strategic traders.*

[11] I should have written $\bar{\beta}_{ND}^0$ in the zero-endowment case of the previous section. I use the same notations in this section, by a slight abuse of notation.

- *The discount decreases faster when n is small.*

4.1.2 Predatory trading

Price schedule. When hedgers anticipate distress at time 1, the price schedule is:

$$p_0^d\left(\sum_{j=2}^{n} x_0^j, x_0^1\right) = D - \beta\frac{n+1}{n}X_{-1}^0 - \beta\frac{1}{n}X_{-1}^1 + \beta\frac{n+1}{n}\sum_{i=2}^{n} x_0^i + \beta x_0^1 \tag{4.6}$$

Equation (4.6) generalizes equation (3.7) to the case with positive hedgers' endowment. The constant of the price schedule decreases when hedgers have positive endowment, leading to the following comparative static:

Corollary 5 *The price decreases more ahead of an anticipated firesale when hedgers have a larger endowment in the risky asset.*

The intuition is that the hedgers now have a lower marginal valuation for the asset and are thus more eager to offload their risk ahead of the prey's firesale.

Equilibrium. The predatory trading equilibrium strategies are built as in the previous section (equations (3.11)- (3.12)). A new variable $\theta = \frac{X_{-1}^0}{X_{-1}^1}$ appears in the equilibrium condition: it measures the strength of the risk-sharing motive (higher X_{-1}^0) relative to the potential benefit of a firesale (given by X_{-1}^1).

Proposition 5 *There exists a predatory trading equilibrium in which the prey is distressed iff $\beta \in I_P$, where I_P is as follows:*

- *If $a \geq \max\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, then $I_P = \left[\underline{\beta}_d \wedge \beta_F, \beta_F\right[$*
- *If $a \leq \min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, then $I_P = \left[\underline{\beta}_d, \bar{\beta}_D\right[$*
- *If $\min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right) < a < \max\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, then*
  - *If $\theta > \theta^*$, then $I_P = \left[\underline{\beta}_d \wedge \beta_F, \underline{\beta}_{d,2} \wedge \beta_F\right[$,*
  - *If $\theta \leq \theta^*$, then $I_P = \left[\underline{\beta}_d \wedge \bar{\beta}_d, \bar{\beta}_d\right[$.*

with $\underline{\beta}_d$ and $\underline{\beta}_{d,2}$ are given by equations (C.22)-(C.23).

The equilibrium price is:

$$p_0 = \bar{p}_0 \tag{4.7}$$

$$p_1 = D + \varepsilon_1 - \beta\frac{\bar{X}}{n+1} - \frac{|R|}{n+1} \tag{4.8}$$

Proposition 5 shows that the equilibrium is driven by three factors: the prey's leverage capacity, a, the ratio $\theta = \frac{X^0_{-1}}{X^1_{-1}}$, and the number of predators (since the coefficients m_1, m_2, κ_2 are functions of n)[12] . Intuitively, θ measures the selling pressure caused by hedgers' willingness to share risk relative to that caused by the prey's firesale. The result suggests that predatory trading can occur in equilibrium whether θ is large relative to a or not, i.e. θ plays an ambiguous role.

4.2 Implications

4.2.1 Hedgers' endowment and probability of predatory trading

Using the results of Proposition 5, I can calculate the probability of predation. The "gross" probability is unadjusted for the fact that the liquidity provision equilibrium can coexist with the predatory trading equilibrium. The "net" probability does take into account the possible coexistence of equilibria. I obtain the following comparative statics with respect to θ.

Corollary 6 *The gross and net probabilities of predation vary as follows.*

- *If $a \leq \min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, denote $\kappa = \frac{\theta+1}{a}$ and define the gross probability of predatory trading $\hat{q}$ as*

  $$\hat{q}(\kappa, n) = \frac{\bar{\beta}_d - \underline{\beta}_d}{\bar{\beta}_d}$$

  $\hat{q}$ decreases in κ, i.e. $\hat{q}$ decreases with θ on this interval.

- *If θ is small, such that $a \geq \max\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, the equilibrium thresholds are ordered as follows: $\underline{\beta}_{nd} < \beta_F < \bar{\beta}_d \wedge \bar{\beta}_d$. Hence the net probability of predation q is*

[12] Note that $\forall n \geq 2$, $\frac{1}{\kappa_2} \leq 1$, and $\max\left(m_2, \frac{1}{\kappa_2}\right) = m_2$. Hence, given that $a \geq 1$, in the special case $X^0_{-1} = 0$, i.e. $\theta = 0$, the equilibrium condition is $\beta \in \left[\underline{\beta}_d \wedge \beta_F, \beta_F\right[$, as in Proposition 1.

given by

$$q(\theta, n, a) = 1 - \frac{\underline{\beta}_{nd}}{\beta_F}$$

Then for θ small, q increases with θ.

The effect of θ on the probability of predation is non-monotonic[13]. If the hedgers' initial positions relative to the prey's are sufficiently large, then increasing θ decreases the likelihood of predatory trading. However, if θ is initial small, then increasing it may *increase* the probability of predatory trading. There are two conflicting effects. First, hedgers' initial position determines the equilibrium illiquidity discount. A high discount makes it easier to push the prey into distress. Second, a large endowment raises the opportunity cost of pushing the prey into distress. This is because predatory trading aims at decreasing the price at which strategic traders can buy the asset. However, if the price is already low because hedgers have large positions to offload, there is a low incentive to engage in predatory trading.

4.2.2 Liquidity hoarding or selling?

In Corollary 5, I showed that when X_1^0 is large, there is a larger price discount at time 0. Interestingly, this effect may be so strong that the predators may not have to sell the asset to trigger the prey's distress.

Corollary 7 (Need to Short-sell) *The following holds:*

- *If $\bar{X} > \frac{n+1}{n}S$ (strong prey), predators* must *go short to trigger the prey's distress.*
- *If $\bar{X} \leq \frac{n+1}{n}S$ (weaker prey), predators need not short-sell for β large enough.*

If the prey is weaker and the market illiquid enough (high β), predatory do not need to short-sell to trigger distress. In that case only hedgers unwind their position and it suffices to push the prey into distress. Predators stay on the sideline, hoading liqudity.

This result has two implications. First, it implies that short-selling ban may be ineffective in curbing predatory trading. The use of short-selling bans was widespread in the 2007-2009 crisis, often to avoid cascades of bank failures (Beber and Pagano, 2012). Second, the result implies that predatory trading is probably under-identified in the data: empirical studies

[13] I checked numerically the "net" probability of predation, i.e. taking into account equilibrium overlap, has typically the same properties as the "gross" probability.

usually consider large funds as potential predators and among those, identify 'predators' as the short-selling ones. Instead, the result shows that distress may occur without short-selling (neither by large funds, nor by the rest of the market, which is only selling their holdings). By contrast, in a model with long-term value traders, predators must always sell or short-sell to induce distress.

4.2.3 Price effects

Predatory trading involves a price manipulation in the first period in order to push the prey into distress. Therefore the illiquidity discount is larger than in the no-distress case at time 0 when predators engage in predatory trading. The price effects of predatory trading at time 1 are as follows:

Corollary 8 *In the equilibrium with distress,*

- *The illiquidity discount at $t = 1$ is larger when the prey has a larger capacity, $\frac{\partial \Gamma_1}{\partial \bar{X}} < 0$, and when the prey has more cash or a less severe constraint $\underline{V}$, $\frac{\partial \Gamma_1}{\partial |R|} < 0$.*
- *The price rebounds on average at $t = 1$ and the average rebound is stronger when the prey is less exposed to forced liquidations (e.g. has more cash, or a looser constraint $\underline{V}$), $\frac{E_0(p_1 - p_0)}{\partial |R|} > 0$, and stronger if the prey has a smaller capacity, $\frac{E_0(p_1 - p_0)}{\partial \bar{X}} < 0$.*

If the prey has a large capacity constraint, there is a large firesale at time 1, hence a large discount and a low price rebound, on average. A lower distress threshold $\bar{p}_0$ is lower leads to a lower time-1 price, but the average rebound is larger. This is because decreasing the price involves to take low or short positions at time 0, therefore predators must buy more aggressively at time 1, leading to a higher rebound on average.

5 Conclusion

I study predatory trading in a model where competitive investors (hedgers) are rational and choose their demand optimally and where the prey's distress in endogenous. This is in contrast with most of the literature, which relies on exogenous demand curves and / or exogenous distress. I show that hedgers' endogenous reactions to the possibility of predatory trading can make predation cheaper. This reaction manifests itself through a change in market depth, which allows predators to move prices more easily than the prey and increases downward pressure on the price. An important determinant of predatory trading is hedgers' risk-bearing capacity. Risk-baring capacity determines hedgers' ability to take the other side of predatory trades and eventually to absorb firesales without causing large market disruptions. The model yields new predictions about individual traders' price impact and the usefulness of trading restrictions such as short-selling bans.

Appendix

The following proofs are given in the case where the hedgers' endowment is $X^0_{-1} \geq 0$. Section D of this appendix contains additional derivations related to the special case where the hedgers have no endowment ($X^0_{-1} = 0$).

A Time-1 subgame and time-0 price schedules

I start by providing two intermediary results about the subgame equilibrium at time 1 and the ensuing price schedules at time 0.

A.1 Time-1 subgame equilibrium

The following result includes Lemma 1 as a special case.

Lemma 7 (Time-1 subgame) *In the time-1 subgame, the trade and payoff are given by*

$$\forall i = 1, ..., n,\ x^i_1 = \frac{S - \sum_{j=1}^n X^j_0}{n+1} \tag{A.1}$$

$$\pi^{i,nd}_1\left(\sum_{j=2}^n x^j_0, x^1_0\right) = \beta \frac{\left(S - \sum_{j=1}^n X^j_0\right)^2}{(n+1)^2} \tag{A.2}$$

when there is no distress and by

$$x^1_1 = -X^1_0 \tag{A.3}$$

$$\forall i = 2, ..., n,\ x^i_1 = \frac{S - \sum_{j=2}^n X^j_0}{n} \tag{A.4}$$

$$\pi^{i,d}_1\left(\sum_{j=2}^n x^j_0, x^1_0\right) = \beta \frac{\left(S - \sum_{j=2}^n X^j_0\right)^2}{(n)^2}, \quad i = 2, \ldots, n \tag{A.5}$$

$$\pi^{1,d}_1\left(\sum_{j=2}^n x^j_0, x^1_0\right) = -\beta X^1_0 \frac{S - \sum_{j=2}^n X^j_0}{n}, \tag{A.6}$$

when the prey is distressed.

Proof Starting from hedgers' demand given by (2.1), inverting the demand curve, and imposing market-clearing implies that

$$\forall t = 0, 1, \; S = X_t^0 + \sum_{j=1}^{n} X_t^j, \tag{A.7}$$

which gives the price schedule faced by strategic traders:

$$p_1\left(\sum_{j=1}^{n} x_1^j\right) = D_1 - \beta\left(S - \sum_{j=1}^{n} X_1^j\right)$$

Using $X_t^j = X_{t-1}^j + x_t^j$ gives:

$$p_1\left(\sum_{j=1}^{n} x_1^j\right) = D_1 - \beta\left(S - \sum_{j=1}^{n} X_0^j\right) + \beta\sum_{j=1}^{n} x_1^j \tag{A.8}$$

There are two states of the world at $t = 1$, with and without distress. If there is distress, the prey must liquidate her entire portfolio, i.e. $X_1^1 = 0$, which implies $x_1^1 = -X_0^1$. Otherwise, the prey is free to choose her position.

- *First case: no distress (nd).* A strategic trader's value function is defined as

$$\forall i = 1, ..., n, \; J_1^{i,nd} \quad = \quad \max_{x_1^i} \mathbb{E}_1\left[B_0^i - x_1^i p_1(\sum_{j=1}^{n} x_1^j) + X_1^i \tilde{D}_2\right]$$

Plugging the price schedule in the maximand gives:

$$\forall i = 1, ...n, \; J_1^{i,nd} = \max_{x_1^i} B_0^i + X_0^i D_1 + x_1^i\left[S - \sum_{j=1}^{n} X_0^j - \sum_{j \neq i}^{n} x_1^j - x_1^i\right],$$

where, $\forall j = 1, ..., n, \; X_0^j$ has been determined in the previous period. Taking the first-order condition, solving for its zero and rearranging terms, we get:

$$\forall i = 1, ..., n, \; x_1^i + \sum_{j=1}^{n} x_1^j = S - \sum_{j=1}^{n} X_0^j \tag{A.9}$$

Collecting the n equations and using matrix notation gives

$$(I+\mathbf{1})\,.\mathbf{x_1} = \left(S - \sum_{j=1}^{n} X_0^j\right).1,$$

where $\mathbf{1}$ is a (n,n) matrix of 1's, $\mathbf{x_1} = (x_1^1, ..., x_1^n)$ and 1 is a vector of 1's. The lines and columns of the matrix $A = I + \mathbf{1}$ are linearly independent. Thus the matrix is invertible with inverse A^{-1} and multiplying on both sides from the left by A^{-1} gives the unique equilibrium in the subgame given by equation (A.1). Plugging this subequilibrium trade into the strategic trader's value function $J_1^{i,nd}$ gives

$$J_1^{i,nd} = B_0^i + X_0^i D_1 + \beta \frac{\left(S - \sum_{j=1}^{n} X_0^j\right)^2}{(n+1)^2} \tag{A.10}$$

The strategic trader's value function is the expected payoff on his date 0 positions in the riskfree and risky assets, plus the continuation payoff given by equation (A.2). When hedgers have no endowment, it suffices to use (A.7), evaluated at $t = -1$ (accounting identity) to obtain the expression in the text.

- *Second case: prey is in distress (d).* In this case, $X_1^1 = 0$, hence $x_1^1 = -X_0^1$. Given that $X_1^1 = 0$, the problem of a predator is

$$\forall i = 2, ..., n,\ \ J_1^{i,d} = \max_{x_1^i} \mathbb{E}_1 \left(B_0^i - x_1^i p_1 \left(\sum_{i=2}^{n} x_1^i \right) + X_1^i \tilde{D}_2 \right)$$

Repeating the same steps as above, I get the unique equilibrium in the subgame, given by equations (A.3)-(A.4). Strategic trader's value function are then given by:

$$\forall i = 2, ..., n,\ \ J_1^{i,d} = B_0^i + X_0^i D_1 + \beta \frac{\left(S - \sum_{j=2}^{n} X_0^j\right)^2}{n^2} \tag{A.11}$$

$$J_1^{1,d} = B_0^1 + X_0^1 D_1 - \beta X_0^1 \frac{S - \sum_{j=2}^{n} X_0^j}{n} \tag{A.12}$$

Thus we can define $\pi_1^{i,d}$ and $\pi_1^{i,d}$ by equations (A.6)-(A.5). As before, when hedgers have no endowment, it suffices to use (A.7), evaluated at $t = -1$ (accounting identity) to obtain the expression in the text.

■

A.2 Time-0 price schedules

In this section, I derive equations (4.1) and (4.6) given in the text. These equations include Lemma 2 as a special case.

Proof I solve for the price schedule at date 0, depending on hedgers' beliefs about the state at $t = 1$.

- *First case*: hedgers believe that the prey will be solvent at $t = 0$. Inverting hegders' demand (2.1) and imposing market-clearing (equation (A.7)) gives

$$p_0^{nd}\left(\sum_{j=2}^{n} x_0^j, x_0^1\right) = D - \beta\frac{n+2}{n+1}\left[S - \sum_{j=1}^{n} X_0^j\right]$$

Using equation (A.7) gives the date-0 price schedule when hedgers anticipate no distress:

$$p_0^{nd}\left(\sum_{j=2}^{n} x_0^j, x_0^1\right) = D - \beta\frac{n+2}{n+1}X_{-1}^0 + \beta\frac{n+2}{n+1}\sum_{j=1}^{n} x_0^j \tag{A.13}$$

With $X_{-1}^0 = 0$, equation (A.13) corresponds to equation (3.6) given in the text.

- *Second case*: Suppose that hedgers believe the prey will be in distress at $t = 1$. From hedgers' demand (2.1) and (A.7) leads to

$$p_0^{d}\left(\sum_{j=2}^{n} x_0^j, x_0^1\right) = D - \beta\frac{n+1}{n}X_{-1}^0 + \beta\sum_{j=1}^{n} x_0^j + \beta\frac{1}{n}\left(\sum_{j=1}^{n} x_0^j - X_0^1\right)$$

Strategic traders' identities are public information, hence, using the dynamics of asset holdings, $X_0^1 = X_{-1}^1 + x_0^1$, this equation can be rewritten as:

$$p_0^{d}\left(\sum_{j=2}^{n} x_0^j, x_0^1\right) = D - \beta\frac{n+1}{n}X_{-1}^0 - \beta\frac{1}{n}X_{-1}^1 + \beta\frac{n+1}{n}\sum_{j=2}^{n} x_0^i + \beta x_0^1 \tag{A.14}$$

Setting $X_{-1}^0 = 0$ gives equation (3.7) in the text. ■

B Liquidity provision equilibrium

To obtain the liquidity provision equilibrium, I first compute the equilibrium in the absence of constraints. Then in a series of auxiliary results, I provide conditions under which the prey and predators do not deviate from this equilibrium despite the presence of constraints.

B.1 Equilibrium without constraint

Lemma 8 (Liquidity provision without constraint) *In the absence of constraints on the prey, the unique equilibrium at time 0 is characterized by the following trades, prices, and payoff:*

$$\forall i = 1, ..., n,\ x_0^i = \frac{n^2+3n}{n^3+4n^2+3n+2} X_{-1}^0 = c_{0,n} X_{-1}^0 \tag{B.1}$$

$$\forall i = 1, ..., n,\ x_1^i = \frac{n+2}{n^3+4n^2+3n+2} X_{-1}^0 = c_{1,n} X_{-1}^0 \tag{B.2}$$

$$p_0 = D - \beta \frac{(n+2)^2}{n^3+4n^2+3n+2} X_{-1}^0 = D - \beta \rho_{0,n} X_{-1}^0 \tag{B.3}$$

$$p_1 = D_1 - \beta \frac{n+2}{n^3+4n^2+3n+2} X_{-1}^0 = D_1 - \beta \rho_{1,n} X_{-1}^0 \tag{B.4}$$

$$J_0^{nd} = E_{-1}^i + \beta \pi_{0,n} \left(X_{-1}^0\right)^2 \quad \textit{with} \quad \pi_{0,n} = \frac{(n^2+3n+1)(n+2)^2}{(n^3+4n^2+3n+2)^2} \tag{B.5}$$

Proof At date 0, a strategic trader's problem is:

$$J_0^{i,nd} = \max_{x_0^i} \mathbb{E}_0 \left[B_{-1}^i - x_0^i p_0^{nd} \left(\sum_{j=2}^n x_0^j, x_0^1 \right) + X_0^i D_1 + \beta \frac{\left(S - \sum_{j=1}^n X_0^j\right)^2}{(n+1)^2} \right] \tag{B.6}$$

Substituting for the price schedule gives

$$J_0^{i,nd} = \max_{x_0^i} E_{-1}^i + \beta \left[\frac{n+2}{n+1} x_0^i \left(X_{-1}^0 - \sum_{j \neq i}^n x_0^j - x_0^i \right) + \frac{\left(X_{-1}^0 - \sum_{j \neq i}^n x_0^j - x_0^i\right)^2}{(n+1)^2} \right]$$

with $E_{-1}^i = B_{-1}^i + X_{-1}^i D$. From the first-order condition, I get:

$$\forall i \in \{1, ..., n\},\ x_0^i + \frac{n^2+3n}{(n+1)^2} \sum_{j=1}^n x_0^j = \frac{n^2+3n}{(n+1)^2} X_{-1}^0 \tag{B.7}$$

Solving this system gives the unique equilibrium in this subgame given by equation (B.1). Substituting this quantity into equation (A.1) gives the date 1 equilibrium trade, given by equation (B.2). Plugging these trades into the price schedules gives the equilibrium prices

given by equations (B.3)-(B.4).
Further, using (B.1) and (B.2), I compute the payoff given by (B.5)
■

B.2 Deviations from liquidity provision in the presence of constraints

Suppose that hedgers believe that the prey will not be distressed. Since the hegders are rational, their beliefs must be correct in equilibrium. I now determine under which condition strategic traders' actions are consistent with hedgers' beliefs.
Problem (B.6) now includes the prey's constraints:

$$J_0^{i,nd} = \max_{x_0^i} \mathbb{E}_0 \left[B_{-1}^i - x_0^i p_0^{nd} \left(\sum_{j=2}^n x_0^j, x_0^1 \right) + X_0^i D_1 + \beta \frac{\left(S - \sum_{j=1}^n X_0^j \right)^2}{(n+1)^2} \right] \tag{B.8}$$

$$\text{s.t. } B_0^1 + X_0^1 p_0 \leq \underline{V} \Rightarrow X_1^1 = 0$$

$$X_0^1 \leq \bar{X} \tag{B.9}$$

I conjecture that the equilibrium trade is given by equation (B.1). The trade satisfies Assumption 2 if $X_{-1}^1 + c_{0,n} X_{-1}^0 \leq \bar{X}$, so I will maintain this assumption throughout.
In the presence of the financial constraints, one must check for two types of deviations. First, the prey may opt for a voluntary liquidation. Second, a strategic trader may turn predator and exploit the prey's constraints to trigger a forced liquidation.

B.2.1 Prey's incentives to deviate from liquidity provision

Lemma 9 (Prey's deviation from liquidity provision) *If predators trade the liquidity provision trade given by equation (B.1), the prey has no incentive to deviate.*

Proof If the prey liquidates, she internalizes that she will change her continuation payoff to $\pi_1^{1,d}$ as long as the price falls below $\bar{p}_0$, i.e., the prey's optimal voluntary liquidation strategy maximizes

$$E_{-1}^1 + x_0^1 \left(D - p_0^{nd} \left(\sum_{j=2}^n x_0^j, x_0^1 \right) \right) + \pi_1^{1,d} \left(\sum_{j=2}^n x_0^j, x_0^1 \right)$$

subject to Assumptions 1 and 2 and x_0^i given by equation (B.1) for $i = 2, \ldots, n$. From the first-order condition of the prey's problem, we get

$$x_0^{1,ovl} = \frac{n^2+n-1}{2n(n+2)}\left(S - \sum_{j=2}^{n} X_0^j\right) - \frac{1}{2}X_{-1}^1$$

Let's ignore first the constraints $p_0 \leq \bar{p}_0$ and $X_0^1 \leq \bar{X}$. When predators trade the conjectured liquidity provision strategy (B.1),

$$S - \sum_{j=2}^{n} X_0^j = \frac{2(n^2+3n+1)}{n^3+4n^2+3n+2}X_{-1}^0 + X_{-1}^1$$

Substituting, we get

$$x_0^{1,ovl} = \frac{(n^2+n-1)(n^2+3n+1)}{n(n+2)\phi_n}X_{-1}^0 - \frac{n+1}{2n(n+2)}X_{-1}^1$$

where $\phi_n \equiv n^3+4n^2+3n+2$. This implies that

$$S - \sum_{j=1}^{n} X_0^j = \frac{(n^2+3n+1)^2}{n(n+2)\phi_n}X_{-1}^0 + \frac{n+1}{2n(n+2)}X_{-1}^1$$

These expressions imply the following payoff for an optimal voluntary liquidation

$$\begin{aligned} J_0^{ovl} = E_{-1}^i &+ \beta\frac{(n^2+3n+1)^2(n^2+n-1)^2}{n^2(n+1)(n+2)\phi_n^2}\left(X_{-1}^0\right)^2 \\ &- \beta\frac{2n^2+5n+1}{4n^2(n+2)}\left(X_{-1}^1\right)^2 - \beta\frac{(n^2+3n+1)(3n^2+5n-1)}{n^2(n+2)\phi_n}X_{-1}^1X_{-1}^0 \end{aligned} \tag{B.10}$$

Comparing J_0^{ovl} to J_0^{nd} (B.5) shows that $J_0^{ovl} < J_0^{nd}$, as X_{-1}^0 and X_{-1}^1 are positive. Once we take into account the constraints, all the payoffs will be lower than J_0^{ovl}, therefore it is never in the interest of the prey to liquidate voluntarily when predators provide liquidity. ■

B.2.2 Predators' incentives to deviate from liquidity provision

Lemma 10 (No self-fulfilling distress deviation) *Suppose that predators (except i) and the prey trade the liquidity provision quantity (B.1). There is no self-fulfilling distress if and only if $\beta < \bar{\beta}_{nd}$, where $\bar{\beta}_{nd}$ is given by equation (B.11). On this parameter interval, inducing distress requires a deviating predator to trade the amount given by equation (B.12) to push* *the price to $\bar{p}_0$. The payoff of this deviation is given by equation (B.14).*

This result includes **Lemma 3** as a special case.

Proof Suppose that $n-2$ predators and the prey trade the liquidity provision trade given by equation (B.1) . A deviating predator internalizes that he will change his continuation payoff to $\pi_1^{i,d}$, provided the price falls below $\bar{p}_0$. Using the expressions for p_0^{nd} and π_1^d, the payoff from exploiting the prey's financial constraints for predator i is:

$$J_0^{i,nd,dev} = \max_{x_0^i} E_{-1}^i + \beta \left[\frac{n+2}{n+1} x_0^i \left(S - \sum_{j=1}^{n} X_0^j \right) + \frac{\left(S - \sum_{j=2}^{n} X_0^j \right)^2}{n^2} \right]$$
$$\text{s.t. } p_0 \leq \bar{p}_0$$

where $i \in \{2, ..., n\}$ and x_0^j given by equation (B.1) for $j = 1, \ldots, n$, with $j \neq i$. Using (A.7), this problem can be rewritten as

$$\max_{x_0^i} \ \beta \frac{n+2}{n+1} x_0^i \left[X_{-1}^0 - \sum_{j=1, j\neq i}^{n} x_0^j - x_0^i \right] + \beta \frac{\left[X_{-1}^0 + X_{-1}^1 - \sum_{j=2, j\neq i}^{n} x_0^j - x_0^i \right]^2}{n^2}$$
$$\text{s.t. } p_0 \leq \bar{p}_0$$

Solving for the zero of the first-order condition, the solution of the unconstrained problem is:

$$x_0^{i,dev} = \frac{n^5 + 5n^4 + 4n^3 - 10n^2 - 11n - 2}{(n^3 + 2n^2 - n - 1)\,\phi_n} X_{-1}^0 - \frac{n+1}{n^3 + 2n^2 - n - 1} X_{-1}^1$$

As a consequence,

$$\frac{n+2}{n+1} \left[X_{-1}^0 - \sum_{j=1}^{n} x_0^j \right] = H_1 X_{-1}^0 + H_2 X_{-1}^1$$

with $H_1 = \frac{n(n+2)\left(n^4+5n^3+8n^2+6n+3\right)}{(n+1)(n^3+2n^2-n-1)\phi_n}$ and $H_2 = \frac{n+2}{n^3+2n^2-n-1}$. This, in turn, implies that $p_0 \leq \bar{p}_0$ iff

$$\beta \geq \bar{\beta}_{nd} = \frac{|R|}{H_1 X_{-1}^0 + H_2 X_{-1}^1}, \text{with } R = \bar{p}_0 - D \tag{B.11}$$

Therefore, I will now focus on the parameter space $\beta < \bar{\beta}_{nd}$. On this interval, pushing the prey into distress requires for a predator to set:

$$p_0^{nd} = \bar{p}_0$$

That is, predator i must choose $x_0^{i,dev}$ such that

$$D - \beta\frac{n+2}{n+1}X_{-1}^0 + \beta\frac{n+2}{n+1}\sum_{j=1,j\neq i}^{n} x_0^j + \beta\frac{n+2}{n+1}x_0^{i,dev} = \bar{p}_0$$

where $\forall j \neq i, x_0^j = c_{0,n}X_{-1}^0$. Rearranging the terms, I get:

$$x_0^{i,dev} = \frac{n+1}{n+2}\frac{R}{\beta} + \frac{2\left(n^2+3n+1\right)}{n^3+4n^2+3n+2}X_{-1}^0 \tag{B.12}$$

Based on this trade, I calculate the payoff of a predator deviating from liquidity provision. First, note that

$$X_{-1}^0 + X_{-1}^1 - \sum_{j=2}^{n} x_0^j = X_{-1}^1 + \frac{n^2+3n}{n^3+4n^2+3n+2}X_{-1}^0 - \frac{n+1}{n+2}\frac{R}{\beta} \tag{B.13}$$

Therefore, using equations (B.12) and (B.13), and developping and rearranging terms, predator i gets the following payoff from deviating and pushing the prey into distress:

$$\begin{aligned} J_0^{i,nd,dev} = {} & E_{-1}^i + \beta\frac{(n+3)^2}{\left(n^3+4n^2+3n+2\right)^2}\left(X_{-1}^0\right)^2 \\ & +\beta\left[\frac{1}{n^2}\left(X_{-1}^1\right)^2 + \frac{2(n+3)}{n\left(n^3+4n^2+3n+2\right)}X_{-1}^1X_{-1}^0\right] \\ & -R\left[\frac{2\left(n^4+5n^3+8n^2+6n+3\right)}{n(n+2)\left(n^3+4n^2+3n+2\right)}X_{-1}^0 + \frac{2(n+1)}{n^2(n+2)}X_{-1}^1\right] \\ & -\frac{(n+1)\left(n^3+2n^2-n-1\right)}{n^2(n+2)^2}\frac{R^2}{\beta} \end{aligned} \tag{B.14}$$

■

B.3 Liqudity provision equilibrium with constraints

In this section, I prove **Proposition 4** given in the text.

Proof Building on Lemma 10 and using equations (B.5) and (B.14), predator i prefers liquidity provision over preying iff $J_0^{i,nd} \geq J_0^{i,nd,dev}$. This condition is equivalent to:

$$\begin{aligned} a_{nd}\beta^2 + b_{nd}\beta + c_{nd} &\geq 0 && \text{(B.15)} \\ \text{where } a_{nd} &= \lambda_1 \left(X^0_{-1}\right)^2 - \lambda_2 \left(X^1_{-1}\right)^2 - \lambda_3 X^1_{-1} X^0_{-1} && \text{(B.16)} \\ b_{nd} &= R\left[\lambda_4 X^0_{-1} + \lambda_5 X^1_{-1}\right] < 0 && \text{(B.17)} \\ c_{nd} &= \lambda_6 R^2 > 0 && \text{(B.18)} \end{aligned}$$

with

$$\lambda_1 = \frac{n^4 + 7n^3 + 16n^2 + 10n - 5}{\left(n^3 + 4n^2 + 3n + 2\right)^2}, \quad \text{(B.19)}$$

$$\lambda_2 = \frac{1}{n^2}, \quad \text{(B.20)}$$

$$\lambda_3 = \frac{2\left(n+3\right)}{n\left(n^3 + 4n^2 + 3n + 2\right)}, \quad \text{(B.21)}$$

$$\lambda_4 = \frac{2\left(n^4 + 5n^3 + 8n^2 + 6n + 3\right)}{n\left(n+2\right)\left(n^3 + 4n^2 + 3n + 2\right)}, \quad \text{(B.22)}$$

$$\lambda_5 = \frac{2\left(n+1\right)}{n^2\left(n+2\right)}, \quad \text{(B.23)}$$

$$\lambda_6 = \frac{\left(n+1\right)\left(n^3 + 2n^2 - n - 1\right)}{n^2\left(n+2\right)^2}. \quad \text{(B.24)}$$

Note that for all $k = 1, ..., 6,$, for all $n \geq 2$, $\lambda_k > 0$. The discriminant of the LHS of inequality (B.15) is

$$\Delta_{nd} = R^2\left[A_1\left(X^0_{-1}\right)^2 + A_2\left(X^1_{-1}\right)^2 + A_3 X^1_{-1} X^0_{-1}\right] \quad \text{(B.25)}$$

with $A_1 = \lambda_4^2 - 4\lambda_1\lambda_6 = \frac{4\left(3n^6+39n^5+104n^4+170n^3+125n^2+36n+4\right)}{n^2(n+2)^2(n^3+4n^2+3n+2)^2} > 0$, $A_2 = \lambda_5 + 4\lambda_6\lambda_2 > 0$, $A_3 = 2\lambda_4\lambda_5 + 4\lambda_6\lambda_3 > 0$. Hence for all $n \geq 2$, $\Delta_{nd} > 0$, which guarantees that there are always two real roots, β_1, β_2. Since the sign of b_{nd} and c_{nd} is known, the sign of equation (B.15) depends on the sign of a_{nd}.

Using $\theta = \frac{X^0_{-1}}{X^1_{-1}}$, I rewrite equation (B.16) as

$$a_{nd} = \left(X^1_{-1}\right)^2\left[\lambda_1\theta - \lambda_3\theta - \lambda_2\right]$$

The discriminant of the equation in parenthesis is $\Delta_a = \lambda_3^2 + 4\lambda_1\lambda_2 > 0$. Since $\lambda_1 > 0$ and

$-\lambda_2 < 0$, there is a positive and a negative root. The positive root is given by

$$\bar{\theta} = \frac{\lambda_3 + \sqrt{\Delta_a}}{2\lambda_1}$$

and since $\theta \geq 0$, the sign of a_{nd} is striclty negative iff $\theta \in \left[0, \bar{\theta}\right[$ and positive iff $\theta > \bar{\theta}$.
I can now determine the equilibrium:

- If $0 \leq \theta < \bar{\theta}$, the no distress equilibrium exists iff $\beta < \beta_1 \wedge \bar{\beta}_{nd}$, with $\beta_1 = -\frac{b_{nd}+\sqrt{\Delta_{nd}}}{2a_{nd}}$.
- If $\theta > \bar{\theta}$, the no distress equilibrium exists iff $\beta < \beta_1 \wedge \bar{\beta}_{nd}$ or $\beta > \beta_2 \wedge \bar{\beta}_{nd}$, with $\beta_2 = \frac{-b_{nd}+\sqrt{\Delta_{nd}}}{2a_{nd}}$.

Using equations (B.16)-(B.18), equation (B.25), and the change of variable $\theta = \frac{X^0_{-1}}{X^1_{-1}}$, the roots are given by

$$\beta_1 = \frac{|R|}{X^1_{-1}} \frac{(\lambda_4\theta + \lambda_5) - \left[A_1\theta^2 + A_3\theta + A_2\right]^{\frac{1}{2}}}{2\left(\lambda_1\theta^2 - \lambda_3\theta - \lambda_2\right)} \equiv \underline{\beta}_{nd} \tag{B.26}$$

$$\beta_2 = \frac{|R|}{X^1_{-1}} \frac{(\lambda_4\theta + \lambda_5) + \left[A_1\theta^2 + A_3\theta + A_2\right]^{\frac{1}{2}}}{2\left(\lambda_1\theta^2 - \lambda_3\theta - \lambda_2\right)} \tag{B.27}$$

I now show that in the second case ($\theta > \bar{\theta}$), the second root, β_2, does not satisfy the parameter restriction $\beta < \bar{\beta}_{nd}$, where $\bar{\beta}_{nd}$ is given by equation (B.11).

Since the denominator of β_2 is strictly positive when $\theta > \bar{\theta}$, $\beta_2 - \bar{\beta}_{nd} < 0$ is, after rearranging terms, equivalent to:

$$\left(\lambda_4 H_1 - 2\lambda_1\right)\theta^2 + \left(\lambda_5 H_1 + \lambda_4 H_2 + 2\lambda_3\right)\theta + \left(\lambda_5 H_2 + 2\lambda_2\right) + \left(H_1\theta + H_2\right)U_\theta^{\frac{1}{2}} < 0$$

where $U_\theta = A_1\theta^2 + A_3\theta + A_2$. Since for all $n \geq 2$, $\lambda_4 H_1 - 2\lambda_1 > 0$ and since all other coefficients are also positive, this condition is never satisfied for any $\theta \geq 0$, hence for any $\theta > \bar{\theta}$. Hence $\beta_2 > \bar{\beta}_{nd}$.

As a result, the necessary and sufficient condition for the existence of the no distress equilibrium is $\beta < \underline{\beta}_{nd} \wedge \bar{\beta}_{nd}$. ■

Corollary 4

Proof The results follow directly from calculations in the proof of Proposition 4. ■

C Predatory trading equilibrium

The conjectured equilibrium predatory trade solves $p_0^d((n-1)x_0^p, x_0^l) = \bar{p}_0$, which implies $\forall j = 2, ..., n$,

$$x_0^j = x_0^p \equiv \frac{1}{n-1}\left[X_{-1}^0 + X_{-1}^1 + \frac{n}{n+1}\left(\frac{R}{\beta} - \bar{X}\right)\right], \text{with } R = \bar{p}_0 - D \tag{C.1}$$

Using equation (A.4), this implies that their date-1 trade is

$$\forall j = 2, ..., n, \; x_1^j = \frac{1}{n+1}\left(\bar{X} - \frac{R}{\beta}\right) \tag{C.2}$$

which leads to the following price:

$$p_1 = D_1 - \frac{\beta}{n+1}\left(\bar{X} - \frac{R}{\beta}\right)$$

I assume that hedgers believe that the prey will be distressed. I first determine conditions under which the prey's conjectured strategy is optimal given the predators' conjectured strategy.

C.1 Prey's optimal liquidation strategy

In this section, I prove **Lemma 4** given in the text.

Proof The predators' conjectured strategy implies the following first-period price (as a function of the prey's trade):

$$p_0^d((n-1)x_0^p, x_0^1) = \bar{p}_0 - \beta\left[\bar{X} - X_{-1}^1 - x_0^1\right] \tag{C.3}$$

Since the equilibrium strategy is constructed so that the prey can not outbid predators, the prey's problem given predators' trades is to maximise the proceeds of liquidation. Hence the prey's maximisation problem is:

$$\begin{aligned} \max_{x_0^1} \quad E_0 & \quad \left[B_{-1}^1 - x_0^1 p_0^d((n-1)x_0^p, x_0^1) - x_1^1 p_1 + X_1 D_2\right] \\ \text{s.t. } X_1^1 & = 0 \\ x_0^1 & \leq \bar{X} - X_{-1}^1 \\ p_1 & = D_1 - \frac{\beta}{n+1}\left(\bar{X} - \frac{R}{\beta}\right) \end{aligned} \tag{C.4}$$

Plugging the first and last constraints into the maximand, and substituting for p_0^d, this problem can be rewritten as:

$$\max_{x_0^1} B_{-1}^1 - x_0^1 \left[\bar{p}_0 - \beta \left[\bar{X} - X_{-1}^1 - x_0^1\right]\right] + X_0^1 \left[D - \beta \frac{1}{n+1} \left[\bar{X} - \frac{R}{\beta}\right]\right]$$
$$\text{s.t. } x_0^1 \leq \bar{X} - X_{-1}^1$$

Writing the Lagrangian of the problem and solving for the zero of the first-order condition gives:

$$x_0^1 = \begin{cases} \frac{n}{2(n+1)} \frac{|R|}{\beta} + \frac{1}{2}\left[\frac{n}{n+1}\bar{X} - X_{-1}^1\right] & \text{if } \beta > \beta_F \\ \bar{X} & \text{otherwise,} \end{cases}$$

$$\text{where } \beta_F = \frac{|R|}{\frac{n+2}{n}\bar{X} - \frac{n+1}{n}X_{-1}^1} \tag{C.5}$$

Thus, a necessary condition for the conjectured strategy to be an equilibrium is $\beta < \beta_F$. ■

C.2 Ruling out self-fulfilling distress

In this section, I prove **Lemma 5** given in the text.

Proof Using the expressions for p_0^d and $\pi_1^{i,d}$, we can write the problem of Definition 3 as

$$\max_{x_0^i} \ \beta x_0^i \left[\frac{n+1}{n}\left(S - \sum_{j=2}^{n} X_0^j\right) - X_0^1\right] + \beta \frac{\left(S - \sum_{j=2}^{n} X_0^j\right)^2}{n^2}$$
$$s.t. \ p_0 \leq \bar{p}_0, \tag{C.6}$$

which can be rewritten as

$$\max_{x_0^i} \beta x_0^i \left[\frac{n+1}{n}\left(X_{-1}^0 + X_{-1}^1 - \sum_{j=2}^{n} x_0^j\right) - \bar{X}\right] + \beta \frac{\left(X_{-1}^0 + X_{-1}^1 - \sum_{j=2}^{n} x_0^j\right)^2}{n^2}$$
$$s.t. \ p_0 \leq \bar{p}_0$$

After writing the Lagrangian of the problem and solving for the equilibrium, I get:

$$x_0^i = \begin{cases} \frac{1}{n-1}\left[X_{-1}^0 + X_{-1}^1 + \frac{n}{n+1}\left(\frac{R}{\beta} - \bar{X}\right)\right] & \text{if } a > \bar{a}_n \text{ or if } \beta < \bar{\beta}_d \text{ when } a \leq \bar{a}_n \\ \frac{n^2+n-2}{n^3+n^2-2n+2}\left(X_{-1}^0 + X_{-1}^1\right) - \frac{n^2}{n^3+n^2-2n+2}\bar{X} & \text{otherwise} \end{cases}$$

$$\text{with } \bar{a}_n = \frac{\rho_{0,n-1}}{d_n} \tag{C.7}$$

$$\bar{\beta}_d = \frac{|R|}{\rho_{0,n-1}\left(X_{-1}^0 + X_{-1}^1\right) - d_n\bar{X}} \tag{C.8}$$

where $\rho_{0,n-1} = \frac{(n+1)^2}{n^3+n^2-2n+2}$ and $d_n = \frac{n^2-n+2}{n^3+n^2-2n+2}$. Note that symmetry is imposed when the Lagrangian of the constraint is zero, while it is the unique outcome when the constraint is not binding.

Thus, a necessary condition for the conjectured strategy to be a Nash equilibrium is $\beta < \beta_d$ if $a \leq \frac{\rho_{0,n-1}}{d_n}$. ■

C.3 Predatory equilibrium

In this section, I prove **Propositions 2 and 5** given in the text.

Proof The payoff of the conjectured strategy for predators is, using equations (C.1) and (C.2):

$$J_0^{i,D} = E_{-1}^i + \beta\frac{\bar{X}^2}{(n+1)^2} - R\left[\frac{1}{n-1}\left(X_{-1}^0 + X_{-1}^1\right) - \frac{n^2-n+2}{(n-1)(n+1)^2}\bar{X}\right] - \frac{n^2+1}{(n-1)(n+1)^2}\frac{R^2}{\beta} \tag{C.9}$$

Payoff from deviating: "rescuing" the prey. Predator i may not join the predatory attack and "rescue" the prey. All predators are pivotal, hence this rescue implies a change in the continuation payoff from $\frac{S-\sum_{j=2}^n X_0^j}{n^2}$ to $\frac{S-\sum_{j=1}^n X_0^j}{(n+1)^2}$.

The strategy of a deviating predator solves the following problem:

$$
\begin{aligned}
J_0^{i,d,dev} &= \max_{x_0^i} \quad \beta x_0^i \left[\frac{n+1}{n}\left(S-\sum_{j=2} X_0^j\right)-X_0^1\right]+\beta\frac{\left(S-\sum_{j=2}^n X_0^j\right)^2}{(n+1)^2} \\
s.t.\ \forall j\neq i,\ x_0^j &= \frac{1}{n-1}\left[X_{-1}^0+X_{-1}^1+\frac{n}{n+1}\left(\frac{R}{\beta}-\bar{X}\right)\right] \\
X_0^1 &= \bar{X} \\
p_0 &> \bar{p}_0
\end{aligned}
$$

Using equation (A.7), and plugging the first and second constraints into the maximand, the maximisation problem boils down to

$$
\begin{aligned}
J_0^{i,d,dev} &= \max_{x_0^i} \quad \beta x_0^i \left[\frac{n+1}{n(n-1)}\left(X_{-1}^0+X_{-1}^1\right)-\frac{n-2}{n-1}\frac{R}{\beta}-\frac{n+1}{n}x_0^i-\frac{1}{n-1}\bar{X}\right] \\
&+ \quad \frac{\beta}{(n+1)^2}\left[\frac{1}{n-1}\left(X_{-1}^0+X_{-1}^1\right)-\frac{n(n-2)}{n^2-1}\frac{R}{\beta}-\frac{2n-1}{n^2-1}\bar{X}-x_0^i\right]^2 \\
s.t.\ p_0 &> \bar{p}_0
\end{aligned}
$$

Writing the Lagrangian and solving for the first-order condition (ignoring the price constraint for now), I get the strategy of a deviating ("rescuing") predator:

$$
\begin{aligned}
x_0^{i,dev} &= \frac{n^3+3n^2+n+1}{2(n-1)(n^3+3n^2+2n+1)}\left(X_{-1}^0+X_{-1}^1\right) \\
&- \frac{n(n^3+3n^2-n+3)}{2(n^2-1)(n^3+3n^2+2n+1)}\bar{X}-\frac{n(n-2)(n^3+3n^2+n+1)}{2(n^2-1)(n^3+3n^2+2n+1)}\frac{R}{\beta}
\end{aligned} \tag{C.10}
$$

It is easy albeit algebraically tedious to check that $\beta<\bar{\beta}_d$ implies that $p_0>\bar{p}_0$, so that the Lagrangian of the price constraint is always zero.

To compute the payoff of the rescue for predator i, it is convenient to calculate the following quantities:

$$
\frac{n+1}{n}\left(X_{-1}^0+X_{-1}^1-\sum_{j=2}^n x_0^j\right)-\bar{X}=z_1\left(X_{-1}^0+X_{-1}^1\right)-z_2\bar{X}-z_3\frac{R}{\beta} \tag{C.11}
$$

where $z_1=\frac{(n+1)\left(n^3+3n^2+3n+1\right)}{2n(n-1)(n^3+3n^2+2n+1)}$, $z_2=\frac{n^3+3n^2+5n-1}{2(n-1)(n^3+3n^2+2n+1)}$, $z_3=\frac{(n-2)\left(n^3+3n^2+3n+1\right)}{2(n-1)(n^3+3n^2+2n+1)}$. and

$$
\frac{X_{-1}^0-\sum_{j=1}x_0^j}{n+1}=z_1'\left(X_{-1}^0+X_{-1}^1\right)-z_2'\bar{X}-z_3'\frac{R}{\beta} \tag{C.12}
$$

with $z_1' = \frac{n^3+3n^2+3n+1}{2(n^2-1)(n^3+3n^2+2n+1)}$, $z_2' = \frac{3n^4+7n^3+3n^2-3n-2}{2(n-1)(n+1)^2(n^3+3n^2+2n+1)}$, $z_3' = \frac{n(n-2)\left(n^3+3n^2+3n+1\right)}{2(n-1)(n+1)^2(n^3+3n^2+2n+1)}$.
From equations (C.10)-(C.12), skipping some algebra, the payoff of rescuing the prey is:

$$\begin{aligned} J_0^{i,d,dev} &= \beta\left[w_1\bar{X}^2 + w_2\left(X_{-1}^0 + X_{-1}^1\right) - w_3\left(X_{-1}^0 + X_{-1}^1\right)\bar{X}\right] \\ &\quad -R\left[w_4\left(X_{-1}^0 + X_{-1}^1\right) - w_5\bar{X}\right] + w_6\frac{R^2}{\beta} \end{aligned} \tag{C.13}$$

with $w_1 = \frac{n^{10}+9n^9+43n^8+114n^7+155n^6+98n^5+41n^4+50n^3+28n^2-15n+4}{4(n-1)^2(n+1)^4(n^3+3n^2+2n+1)^2}$, $w_2 = \frac{(n+1)^4}{4n(n1)^2(n^3+3n^2+2n+1)}$, $w_3 = \frac{n^6+9n^5+23n^4+24n^3+7n^2+n-1}{2(n-1)^2(n^3+3n^2+2n+1)^2}$, $w_4 = \frac{(n-2)(n+1)^3}{2(n-1)^2(n^3+3n^2+2n+1)}$, $w_5 = \frac{n^2(n-2)\left(n^5+9n^4+23n^3+25n^2+10n+4\right)}{2(n+1)(n-1)^2(n^3+3n^2+2n+1)^2}$, $w_6 = \frac{n(n-2)^2(n+1)^2\left(n^4+4n^3+4n^2+3n+1\right)}{4(n-1)^2(n^3+3n^2+2n+1)^2}$.
The conjectured predatory trades form a Nash equilibrium iff $\forall i = 2, ..., n,\ J_0^{i,d} \geq J_0^{i,d,dev}$. From equations (C.9) and (C.13), this is equivalent to

$$a_d\beta^2 + b_d\beta + c_d \geq 0 \tag{C.14}$$
$$\text{with } a_d = e_1\bar{X}^2 - e_2\left(X_{-1}^0 + X_{-1}^1\right)^2 + e_3\bar{X}\left(X_{-1}^0 + X_{-1}^1\right) \tag{C.15}$$
$$b_d = -R\left[e_4\left(X_{-1}^0 + X_{-1}^1\right) - e_5\bar{X}\right] \tag{C.16}$$
$$c_d = -e_6R^2 \tag{C.17}$$

and $e_1 = \frac{1}{(n+1)^2} - w_1$, $e_2 = w_2$, $e_3 = w_3$, $e_4 = \frac{1}{n-1} - w_4$, $e_5 = \frac{n^2-n+2}{(n-1)(n+1)^2} - w_5$, $e_6 = \frac{n^2+1}{(n-1)(n+1)^2} + w_6$

$$e_4 = \frac{n^4+3n^3+n^2+3n}{2\left(n-1\right)^2\left(n^3+3n^2+2n+1\right)} \tag{C.18}$$
$$e_5 = \frac{n^9-4n^7+24n^6+79n^5+56n^4+14n^3-12n^2-10n-4}{2\left(n-1\right)^2\left(n+1\right)^2\left(n^3+3n^2+2n+1\right)^2} \tag{C.19}$$

It is clear that $c_d < 0$. Let us now study the signs of b_d and a_d.

Sign of b_d

$$b_d \geq 0 \Leftrightarrow \kappa \geq \frac{e_5}{e_4}, \text{ where } \kappa = \frac{X_{-1}^0 + X_{-1}^1}{\bar{X}} \tag{C.20}$$

Further, from equations (C.18)-(C.19), $\forall n \geq 2$, $\frac{e_5}{e_4} = \frac{n^9-4n^7+24n^6+79n^5+56n^4+14n^3-12n^2-10n-4}{(n+1)^2(n^3+3n^2+2n+1)(n^4+3n^3+n^2+3n)}$ and $\frac{e_5}{e_4} \leq 1$.

Sign of a_d

Using the variable $\kappa = \frac{X_{-1}^0 + X_{-1}^1}{\bar{X}}$, I rewrite equation (C.15) as:

$$a_d = \bar{X}^2 \left[e_1 - e_2\kappa^2 + e_3\kappa\right]$$

For $n = 2$, $e_1 < 0$, $e_2 > 0$, $e_3 > 0$. When $n > 2$, all coefficients are strictly positive. Thus,

- If $n = 2$, there are two positive roots, $\kappa_1 = \frac{e_3 - \sqrt{\delta}}{2e_2}$ and $\kappa_2 = \frac{e_3 + \sqrt{\delta}}{2e_2}$, where $\delta = e_3^2 + 4e_2e_1$.
- If $n > 2$, there is a positive and a negative roots, with $\kappa_1 < 0$ and $\kappa_2 > 0$.

Hence, $a_d > 0 \Leftrightarrow$

- $\kappa \in]\kappa_1, \kappa_2[$, if $n = 2$
- $\kappa \in]0, \kappa_2[$, if $n > 2$.

<u>*Discriminant*</u>

The discriminant of equation (C.14) is:

$$\begin{aligned} \Delta_d &= R^2 \left[r_1 \left(X_{-1}^0 + X_{-1}^1\right)^2 + r_2\bar{X}\left(X_{-1}^0 + X_{-1}^1\right) + r_3\bar{X}^2\right] \\ \text{i.e., } \Delta_d &= R^2\bar{X}^2 \left[r_1\kappa^2 + r_2\kappa + r_3\right] \end{aligned} \tag{C.21}$$

with $r_1 = e_4^2 - 4e_6e_2$, $r_2 = 4e_6e_3 - 2e_5e_4$, $r_3 = e_5^2 + 4e_6e_1$. $\forall n \geq 2$, $r_1 > 0$, and $r_2 > 0$. Further, $r_3 < 0$ for $n = 2$ and $r_3 > 0$ for $n > 2$.[14]

Hence if $n = 2$, the equation $r_1\kappa^2 + r_2\kappa + r_3$ has two solutions:

$$\begin{aligned} \kappa_1^d &= \frac{-r_2 + \sqrt{\Delta_d}}{2r_1} \approx 0.1 \\ \kappa_2^d &= \frac{-r_2 - \sqrt{\Delta_d}}{2r_1} < 0, \text{where } \Delta_d = r_2^2 - 4r_1r_3 \end{aligned}$$

If $n > 2$, then all coefficients r_i being strictly positive, $\Delta_D > 0$ for any κ. Hence,

- If $n = 2$, then $\Delta_d < 0$ for $\kappa \in \left[0, \kappa_1^d\right[$. If $\kappa > \kappa_1^d \approx 0.1$, then $\Delta_d > 0$.
- If $n > 2$, then $\Delta_d > 0$.

[14] For the sake of brevity, I did not reproduce the analytical expression of the coefficients r_i. I check the signs numerically for $n = 2$ to $n = 150$.

Equilibrium

The equilibrium is determined by the sign of equation (C.14) and the parameter restrictions β_F and $\bar{\beta}_d$, given by equations (C.5) and (C.8), respectively.

When $\Delta_d > 0$, equation (C.14) has two real roots given by

$$\underline{\beta}_d = \frac{\sqrt{\Delta_d} - b_d}{2a_d} \tag{C.22}$$

$$\underline{\beta}_{d,2} = -\frac{b_d + \sqrt{\Delta_d}}{2a_d} \tag{C.23}$$

It is easy to see that if $a_d > 0$, $\beta_2 < 0$, and if $a_d < 0$, $\beta_2 > \underline{\beta}_d > 0$. Using $\kappa = \frac{X^0_{-1}+X^1_{-1}}{\bar{X}}$, equations (C.22) and (C.23) and (C.15)-(C.17), the roots can be rewritten as:

$$\underline{\beta}_d = \frac{|R|}{\bar{X}} \frac{Z_\kappa^{\frac{1}{2}} - (e_4\kappa - e_5)}{2\left(e_1 - e_2\kappa^2 + e_3\kappa\right)} \tag{C.24}$$

$$\beta_2 = -\frac{|R|}{\bar{X}} \frac{Z_\kappa^{\frac{1}{2}} + (e_4\kappa - e5)}{2\left(e_1 - e_2\kappa^2 + e_3\kappa\right)} \tag{C.25}$$

where $Z_\kappa = r_1\kappa^2 + r_2\kappa + r_3$.

I first study the sign of equation (C.14) independently of the parameter restrictions.

If $n > 2$, $\Delta_d > 0$, hence the equation has two real roots. From the signs of a_d and b_d, there are two thresholds for κ in this case: κ_2 and $\frac{e_5}{e_4}$. Since for all $n \geq 2$, $\kappa_2 \geq 1$ and $\frac{e_5}{e_4} < 1$, it is clear that $\kappa_2 > \frac{e_5}{e_4}$. Then the sign of equation (C.14) is as follows:

- If $\kappa \in \left[0, \frac{e_5}{e_4}\right[$, $a_d > 0$, $b_d < 0$, $c_d < 0$, hence $\beta_2 < 0$, $\underline{\beta}_d > 0$ and $a_d\beta^2 + b_d\beta + c_d \geq 0 \Leftrightarrow \beta > \underline{\beta}_d$
- If $\left[\frac{e_5}{e_4}, \kappa_2\right[$, $a_d > 0$, $b_d > 0$, $c_d < 0$, then $\beta_2 < 0$, $\underline{\beta}_d > 0$ and $a_d\beta^2 + b_d\beta + c_d \geq 0 \Leftrightarrow \beta > \underline{\beta}_d$.
- If $\kappa > \kappa_2$, then $a_d < 0$, $b_d < 0$, and $c_d < 0$ and $a_d\beta^2 + b_d\beta + c_d \geq 0 \Leftrightarrow \beta \in \left[\underline{\beta}_d, \underline{\beta}_{d,2}\right[$

When $n = 2$, there are four thresholds κ_1^d, κ_1, $\frac{e_5}{e_4}$ and κ_2, in increasing order. For $\kappa \geq \frac{e_5}{e_4}$, the analysis is similar to the case where $n > 2$. For $\kappa < \frac{e_5}{e_4}$, the intervals are as follows:

- If $\kappa \in \left[0, \kappa_1^d\right[$, $a_d < 0$, $b_d < 0$, $c_d < 0$, and $\Delta_d < 0$, hence $a_d\beta^2 + b_d\beta + c_d < 0$ and there is no predatory trading equilibrium.

- If $\kappa \in \left[\kappa_1^d, \kappa_1\right[$, then $\Delta_d > 0$, but since $a_d < 0$, $b_d < 0$, $c_d < 0$, there are two negative roots, and therefore, there is no predatory trading equilibrium. This case can be grouped with the previous one.

- If $\kappa \in \left[\kappa_1, \frac{e_5}{e_4}\right[$, then $a_d > 0$, $b_d < 0$, $c_d < 0$ and $\Delta_d > 0$. Then $\beta_2 < 0$, $\underline{\beta}_d > 0$ and $a_d\beta^2 + b_d\beta + c_d \geq 0 \Leftrightarrow \beta > \underline{\beta}_d$. Thus this case can be grouped with the one in which $\kappa > \frac{e_5}{e_4}$.

$\Rightarrow$ The $n = 2$ case is thus the same as the $n > 2$ case, except for $\kappa < \kappa_1$.

I now determine the intervals of the predatory trading equilibrium, taking into account the parameter restrictions β_F and $\bar{\beta}_d$, given by equations (C.5) and (C.22), respectively.

Position of β_F relative to $\bar{\beta}_d$

From equations (C.5) and (C.22):

$$\bar{\beta}_d > \beta_F \Leftrightarrow a \geq m_1\theta + m_2 \tag{C.26}$$

with $m_1 = \frac{n(n+1)^2}{n^4+4n^3-n^2+4}$ and $m_2 = \frac{n^4+3n^3+n^2+n+2}{n^4+4n^3-n^2+4}$

Note that $m_2 = 1$ when $n = 2$ and $m_2 < 1$ when $n > 2$.
$\Rightarrow$ If $\theta = 0$ (i.e. $X^0_{-1} = 0$), $\bar{\beta}_d > \beta_F \Leftrightarrow a \geq m_2$, which is always true since $a \geq 1$.
$\Rightarrow$ Proposition 2 follows from this remark and the analysis below.

Intervals of the predatory trading equilibrium

The analysis of equation (C.14) gives necessary and sufficient conditions in terms of the variable κ, whereas the parameter restrictions for β_F and $\bar{\beta}_d$ are expressed in terms of θ. Noting that[15]:

$$\kappa = \frac{\theta + 1}{a} \tag{C.27}$$

I rewrite all the conditions in terms of a and θ.

The thresholds in terms of κ are κ_1 (for $n = 2$ only), $\frac{e_5}{e_4}$ and κ_2. Hence using equation (C.27), the corresponding thresholds in terms of a are, in increasing order, $\frac{1}{\kappa_1}\theta + \frac{1}{\kappa_1}$, $\frac{e_4}{e_5}\theta + \frac{e_4}{e_5}$ and $\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}$.

[15]Using the definition of κ (C.20) and the following notations: $\theta = \frac{X^0_{-1}}{X^1_{-1}}$, $a = \frac{\bar{X}}{X^1_{-1}}$

I now compare these thresholds to the condition (C.26). For all $n \geq 2$, $\frac{e_4}{e_5} > m_2 > m_1$, $\frac{1}{\kappa_1} > m_2 > m_1$. Therefore, $\forall n \geq 2$,

$$\begin{cases} \frac{e_4}{e_5}\theta + \frac{e_4}{e_5} > m_1\theta + m_2 \\ \frac{1}{\kappa_1}\theta + \frac{1}{\kappa_1} > m_1\theta + m_2 \end{cases}$$

Further, $\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2} > m_1\theta + m_2$ is equivalent to

$$\theta > \theta^* = \frac{m_2 - \frac{1}{\kappa_2}}{\frac{1}{\kappa_2} - m_1}$$

Since $\forall n \geq 2$, $m_2 > \frac{1}{\kappa_2} > m_1$, $\theta^* > 0$. Hence, combining the equilibrium conditions and the parameter restrictions yields, $\forall n > 2$

- If $a \geq \max\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, then $I_P = \left[\underline{\beta}_d \wedge \beta_F, \beta_F\right[$
- If $a \leq \min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, then $I_P = \left[\underline{\beta}_d \wedge \bar{\beta}_d, \underline{\beta}_{d,2} \wedge \bar{\beta}_d\right[$
- If $\min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right) < a < \max\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, then
  - If $\theta > \theta^*$, then $I_P = \left[\underline{\beta}_d \wedge \beta_F, \underline{\beta}_{d,2} \wedge \beta_F\right[$,
  - If $\theta \leq \theta^*$, then $I_P = \left[\underline{\beta}_d \wedge \bar{\beta}_d, \bar{\beta}_d\right[$.

If $n = 2$, there is an additional case: if $a \geq \frac{1}{\kappa_1}\theta + \frac{1}{\kappa_1}$, there is no predatory trading equilibrium.

In the second case, $a \leq \min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, it is possible to refine the boundaries of the interval I_P and show that it is non-empty, thereby proving the existence of the equilibrium in this case.

Existence conditions

I first show that $\underline{\beta}_d < \bar{\beta}_d$. This case is interesting for $a \leq \min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, hence the interval I consider is $\kappa > \kappa_2$. Using (C.24) and (C.8), and rearranging terms, I get

$$\underline{\beta}_d - \bar{\beta}_d = \frac{|R|}{\bar{X}} \frac{g_2(\kappa)}{(\rho_{0,n-1}\kappa - d_n)(e_1 - e_2\kappa^2 + e_3\kappa)} \tag{C.28}$$

$$\text{with } g_2(\kappa) = (\rho_{0,n-1}\kappa - d_n) Z_\kappa^{\frac{1}{2}} + B_1\kappa^2 + B_2\kappa - B_3 \tag{C.29}$$

where $\forall n \geq 2$, $B_1 = 2e_2 - \rho_{0,n-1}e_4 < 0$, $B_2 = e_5\rho_{0,n-1} + d_n e_4 - 2e_3 < 0$, $B_3 = 2e_1 + d_n e_5 > 0$.[16]

The denominator of equation (C.28) is negative when $\kappa > \kappa_2$, thus $\underline{\beta}_d - \bar{\beta}_d < 0$ iff $g_2(\kappa) \geq 0$. To determine the sign of g_2, I first study its first derivative:

$$g_2'(\kappa) = \rho_{0,n-1}Z_\kappa^{\frac{1}{2}} + (\rho_{0,n-1}\kappa - d_n)\frac{Z_\kappa'}{Z_\kappa^{\frac{1}{2}}} + 2B_1\kappa + B_2$$

The first term of the derivative is positive for any $\kappa > 0$. The second term is also positive, because $\forall n \geq 2$, $\frac{d_n}{\rho_{0,n-1}} < \kappa_2$ and $Z_\kappa' = 2r_1\kappa + r_2 > 0$ for any $\kappa > \kappa_2 > 0$ (r_1 and r_2 being positive for any $n \geq 2$). The third term, however, is negative, because B_1 and B_2 are negative. I will show that $\forall \kappa > \kappa_2$, $g_2'(\kappa) > 0$. To show this, it is enough to show that $\rho_{0,n-1}Z_\kappa^{\frac{1}{2}} + 2B_1\kappa + B_2 \geq 0$.

Since $Z_\kappa = r_1\kappa^2 + r_2\kappa + r_3$ (see equation (C.24)), the following holds for any $\kappa > \kappa_2$:

$$Z_\kappa \geq r_1\kappa^2 + r_2\kappa_2 + r_3$$

and therefore $\rho_{0,n-1}\sqrt{Z_\kappa} \geq \rho_{0,n-1}\sqrt{r_1\kappa + r_2\kappa_2 + r_3}$, which implies that

$$\rho_{0,n-1}\sqrt{Z_\kappa} + 2B_1\kappa + B_2 \geq \rho_{0,n-1}\sqrt{r_1\kappa^2 + r_2\kappa_2 + r_3} + 2B_1\kappa + B_2$$

Given that $\forall n \geq 2$, $\rho_{0,n-1}\sqrt{r_1} \geq -2B_1$, the function on the RHS of the inequality is increasing in κ. Hence for $\kappa > \kappa_2$, $\rho_{0,n-1}\sqrt{Z_\kappa} + 2B_1\kappa + B_2 > \rho_{0,n-1}\sqrt{r_1\kappa_2^2 + r_2\kappa_2 + r_3} + 2B_1\kappa_2 + B_2$. The right-hand side of the inequality is positive for all $n \geq 2$, hence $\forall \kappa > \kappa_2$, $\forall n \geq 2$, $g_2'(\kappa) > 0$ and g_2 is increasing on this interval. As a result, one can minor this function by $g_2(\kappa_2)$, with $\forall n \geq 2$, $g_2(\kappa_2) > 0$.

Hence $\forall \kappa > \kappa_2$, $\underline{\beta}_d < \bar{\beta}_d$.

Using a similar reasoning, one can show that $\underline{\beta}_{d,2} > \bar{\beta}_d$ when $\kappa > \kappa_2$. From equations (C.25) and (C.8), $\underline{\beta}_{d,2} < \bar{\beta}_d$ is equivalent to $h_2(\kappa) > 0$, with

$$h_2(\kappa) = -(\rho_{0,n-1} - d_n)\sqrt{Z_\kappa} + B_1\kappa^2 + B_2\kappa - B_3$$

The function $-(\rho_{0,n-1} - d_n)\sqrt{Z_\kappa}$ is always negative, as well as $B_1\kappa^2 + B_2\kappa - B_3$. Thus $\forall \kappa > \kappa_2$, $\underline{\beta}_{d,2} > \bar{\beta}_d$. ■

[16] For the remainder of the proof, I rely again on calculations for the coefficients which are functions of n.

Corollary 6

Proof Suppose that $a \leq \min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, and consider $p(\kappa) = 1 - \hat{q}(\kappa) = \frac{\underline{\beta}_d}{\overline{\beta}_d}$. From equations (C.24) and (C.8), we can write

$$p(\kappa) = \frac{\left(\rho_{0,n-1}\kappa - d_n\right)\left(Z_\kappa^{\frac{1}{2}} - (e_4\kappa - e_5)\right)}{2\left(e_1 - e_2\kappa^2 + e_3\kappa\right)}$$

Hence the first derivative w.r.t. κ, after regrouping terms, is

$$p^{'}(\kappa) = \frac{\left(e_1 - e_2\kappa^2 + e_3\kappa\right)\left(\rho_0 Z_\kappa + (\rho_0 - d_n)(2r_1\kappa + r_2)\right) - (e_3 - 2e_2\kappa)(\rho_0\kappa - d_n) 2Z_\kappa}{2Z_\kappa^{\frac{1}{2}}}$$
$$+ \left(e_5\rho_0 + e_4 d_n - 2e_4\rho_0\kappa\right)\left(e_1 - e_2\kappa^2 + e_3\kappa\right) + (2e_2\kappa - e_3)\left(-e_4\rho_0\kappa^2 (e_5\rho_0 + e_4 d_n)\kappa - d_n e_5\right)$$

It is enough to show that p is increasing when $\kappa \geq \kappa_2$. I start by developing and rearranging terms of the numerator in the first line. Using that $Z_\kappa = r_1\kappa^2 + r_2\kappa + r_3$, I get after a few calculations that the numerator is equal to $H_1\kappa^4 + H_2\kappa^3 + H_3\kappa^2 + H_4\kappa + H_5$, with

$$\begin{aligned}
H_1 &= e_2 r_1 \rho_0;\ H_2 = 2e_2\rho_0 r_2 - 2r_1 d_2 e_2 + r_1 e_3 \rho_0 \\
H_3 &= 3r_3 e_2 \rho_0 - 2r_2 d_n e_2 + 3r_1\rho_0 e_1 + e_3\rho_0 r_2;\ H_4 = 2r_2\rho_0 e_1 - r_3 e_3 \rho_0 - 4r_3 d_n e_2 - e_1 r_1 d_n \\
H_5 &= 2r_1 e_3 d_n - e_1 r_2 d_n
\end{aligned}$$

Now consider the second line in $p^{'}$ and rearrange terms. This gives: $H_6\kappa^2 - H_7\kappa + H_8$, with

$$H_6 = e_2\left(e_5\rho_0 + e_4 d_n\right) + e_3 e_4 \rho_0;\ H_7 = 2e_4 e_1 \rho_0 + 2e_2 d_n e_5;\ H_8 = e_1\left(e_5\rho_0 + e_4 d_n\right) + d_n e_5 e_3$$

Hence the sign of $p^{'}$ is the same as the sign of

$$\phi_\kappa = H_1\kappa^4 + H_2\kappa^3 + H_3\kappa^2 + H_4\kappa + H_5 + 2Z_\kappa^{\frac{1}{2}}\left(H_6\kappa^2 - H_7\kappa + H_8\right)$$

Calculating the coefficients H_i, which are functions of n, we find that H_1, H_2, H_3, H_6 and H_8 are positive for any $n \geq 2$. However, for $n \geq 2$, H_4 is negative, H_7 is positive and H_5 becomes negative for $n \geq 4$. Given the signs of the coefficients, to show that $p^{'}$ is positive for $\kappa \geq \kappa_2$, it is enough to show $H_3\kappa^2 + H_4\kappa + H_5 \geq 0$ and $H_6\kappa^2 - H_7\kappa + H_8$ on this interval.

First, consider $H_3\kappa^2 + H_4\kappa + H_5 \geq 0$. Since $H_3 > 0$, it is increasing for $\kappa \geq -\frac{H_4}{2H_3}$, which calculations show is smaller than κ_2. Further, I find that for any $n \geq 2$, $H_3(\kappa_2)^2 + H_4\kappa_2 + H_5 > 0$. Next, consider $H_6\kappa^2 - H_7\kappa + H_8$ and apply the same steps. H_6 is positive and the function peaks in $\frac{H_7}{2H_6}$, which I find is smaller than κ_2 for $n \geq 2$. Further, I find that

$H_6\kappa^2 - H_7\kappa + H_8 > 0$. As a result, $p^{'}$ is positive for $\kappa \geq \kappa_2$, hence $\hat{q}$ is decreasing on its interval.

■

Corollary 8

Proof Using Proposition 5, we get:

$$E_0\left(p_1 - p_0\right) = D - \bar{p}_0 - \frac{\beta}{n+1}\bar{X} - \frac{|R|}{n+1} = \frac{n|R|}{n+1} - \frac{\beta}{n+1}\bar{X}$$

Thus $E_0\left(p_1 - p_0\right) \geq 0 \Leftrightarrow n|R| - \beta\bar{X} > 0 \Leftrightarrow \beta < \frac{n|R|}{\bar{X}}$. Since$\beta < \beta_F = \frac{n|R|}{(n+2)\bar{X} - (n+1)X_{-1}^1}$, and $\beta_F \leq \frac{n|R|}{\bar{X}} \Leftrightarrow X_{-1}^1 \leq \bar{X}$, we have $E_0\left(p_1\right) \geq p_0$. Clearly, $E_0\left(p_1 - p_0\right)$ increases with —R— and decreases with X_{-1}^1.

The illiquidity discount at time 1, $\Gamma_1 = -\frac{\beta\bar{X} + |R|}{n+1}$. Hence Γ_1 is decreasing in $\bar{X}$ and $|R|$.

■

D Additional derivations for the no trading case

Proposition 1

Proof From Proposition 5, the driver of the equilibrium is the position of a relative to $\max\left(m_2, \frac{1}{\kappa_2}\right)$ and $\min\left(m_2, \frac{1}{\kappa_2}\right)$.

If $X_{-1}^0 = 0$, then $\theta = 0$, and the equilibrium condition simplifies as follows:

- Since $\forall n \geq 2$, $m_2 > \frac{1}{\kappa_2}$ and since $\frac{1}{\kappa_2} \leq 1 \leq a$, the case $a < \min\left(m_2, \frac{1}{\kappa_2}\right)$ does not exist.
- Further, $\forall n \geq 2, m_2 \geq 1$, hence the case $\min\left(m_2, \frac{1}{\kappa_2}\right) < a < \max\left(m_2, \frac{1}{\kappa_2}\right)$ does not exist either.

The only remaining case is thus $a \geq \max\left(m_2, \frac{1}{\kappa_2}\right) = m_2$. Since $\frac{1}{\kappa_2} < m_2 \leq 1$ for all $n \geq 2$, the condition on a is always satisfied. Hence if $\theta = 0$, the equilibrium condition for the equilibrium with predatory trading is $\beta \in \left[\underline{\beta}_d \wedge \beta_F, \beta_F\right[$. ■

Proposition 3

Proof The equilibrium with distress occurs on a non-empty interval iff $\underline{\beta}_d < \beta_F$. Using equations (C.24) and (C.5):

$$\begin{aligned} \underline{\beta}_d - \beta_F &= \frac{|R|}{\bar{X}} f(n,a) \\ \text{with } f(n,a) &= \frac{(u_1 - u_{2a})\left(\sqrt{\gamma_{3a}} - \gamma_{5a}\right) - 2\gamma_{6a}}{2\gamma_{6a}(u_1 - u_{2a})} \end{aligned} \tag{D.1}$$

Similarly, using equations (C.24) and (B.26), I get:

$$\begin{aligned} \underline{\beta}_d - \underline{\beta}_{nd} &= \frac{|R|}{\bar{X}} g(n,a) \\ \text{with } g(n,a) &= \frac{\lambda_2\left(\sqrt{\gamma_{3a}} - \gamma_{5a}\right) - a\gamma_{6a}\left(\sqrt{A_2} - \lambda_5\right)}{2\gamma_{6a}\lambda_2} \end{aligned} \tag{D.2}$$

The no-trading and predatory trading equilibria coexist iff $g(n,a) > 0$. ■

Lemma 6

Proof We can recover Q^d from equation (3.12): $Q^d = \frac{n}{n+1}\frac{R}{\beta} - \frac{n}{n+1}\bar{X} + X^1_{-1}$. Using p_0^{nd} from Lemma 2, $p_0^{nd}\left(Q^{nd}, \bar{X}\right) = \bar{p}_0 \Leftrightarrow Q^{nd} = \frac{n+1}{n+2}\frac{R}{\beta} - \bar{X} + X^1_{-1}$. Thus

$$Q^{nd} \geq Q^d \Leftrightarrow \frac{1}{(n+1)(n+2)}\frac{R}{\beta} \geq \frac{1}{n+1}\bar{X}$$

The left-hand side is strictly negative, while the right-hand side is strictly positive. Hence $Q^{nd} < Q^d$. Further, note that since $\bar{X} > X^1_{-1}$, $Q^{nd} < 0$.
To understand this impact of the change in price schedule on the equilibrium conditions, I redo the analysis of Lemma 4 based on the no-distress price schedule, following identical steps. The prey's problem is

$$\max_{x_0^1, x_0^1 \leq \bar{X} - X^1_{-1}} B^1_{-1} - x_0^1\left[\bar{p}_0 - \beta\frac{n+2}{n+1}\left(\bar{X} - X^1_{-1} - x_0^1\right)\right] + X_0^1\left[D - \frac{\beta}{n+1}\left(\bar{X} - \frac{R}{\beta}\right)\right]$$

I write the Lagrangian of the problem and solve for the zero of the first-order condition (assuming the Lagrangian multiplier is 0). I get:

$$x_0^1 = \frac{n}{2(n+2)}\frac{|R|}{\beta} + \frac{1}{2}\left(\frac{n+1}{n+2}\bar{X} - X^1_{-1}\right)$$

Hence the constraint on the prey's position is not binding if $\frac{n}{2(n+2)}\frac{|R|}{\beta} + \frac{1}{2}\left(\frac{n+1}{n+2}\bar{X} - X^1_{-1}\right) \leq \bar{X} - X^1_{-1}$, which is equivalent to $\beta < \tilde{\beta}_F \equiv \frac{|R|}{\frac{n+3}{n}\bar{X} - \frac{n+2}{n}X^1_{-1}}$. In the proof of Lemma 4, I show that $\beta_F = \frac{|R|}{\frac{n+2}{n}\bar{X} - \frac{n+1}{n}X^1_{-1}}$, hence $\beta_F > \tilde{\beta}_F$.

Similarly, one can predict how the condition for ruling out self-fulfilling distress would change. Since predators have less price impact when the price schedule is p_0^{nd}, it will harder, conditional on distress, to trigger it, thus there should be a larger interval on which predatory trading is not self-fulfilling. In other words, $\tilde{\bar{\beta}}^d > \bar{\beta}^d$. $\blacksquare$

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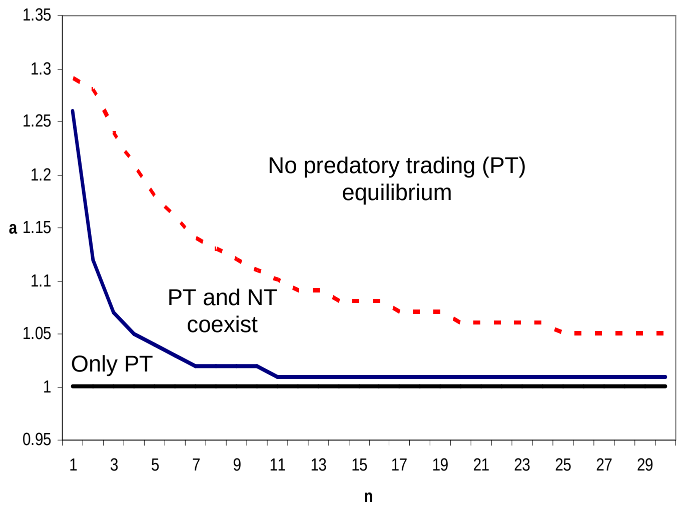
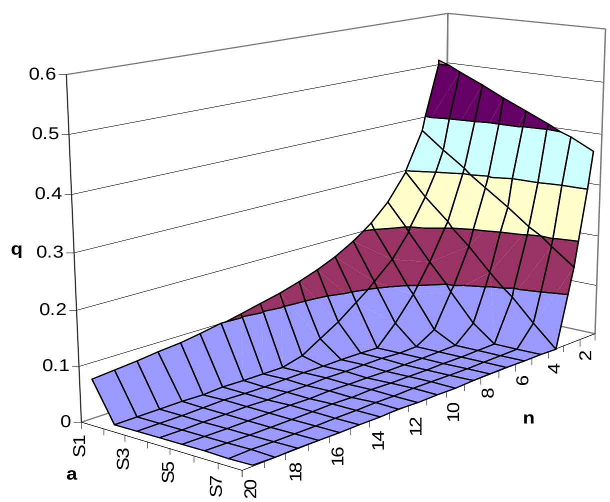
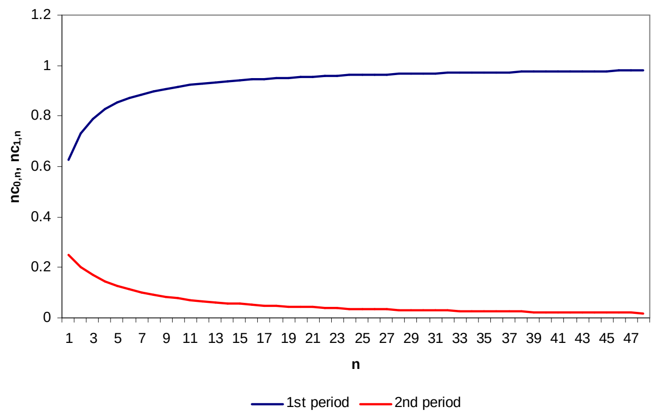


(a) Equilibria with or without distress

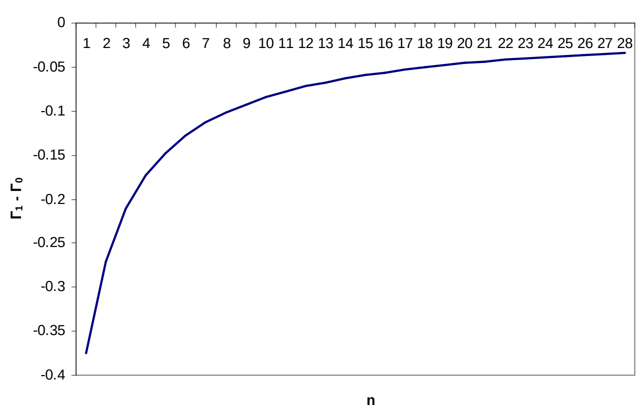


(b) “Net” probability of predatory trading

Figure 1: Coexistence of equilibria and “net” probability of predatory trading as a function of the number of predators n, and the prey’s leverage capacity, $a = \frac{\bar{X}}{X_{-1}^{1}}$. In Panel (b), a varies from 1 (S1) to 1.07 (S7). The calculations assume that β is uniformly distributed between 0 and β_F.



(a) Equilibrium trades



(b) Convergence

Figure 2: Equilibrium trades and speed of convergence of the price towards the fundamental value of the asset as a function of the number of strategic traders in the no-distress equilibrium.

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