Optimal Jury Composition in Contests With Biased Reviewers*

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Abstract

We study the optimal jury composition in a lottery contest game, where each jury member can be biased towards a particular contestant. In our model, a bias in favor of contestant i increases his probability of winning and is the outcome of the voting game between the jury members. We show that the optimal jury composition strongly depends on the degree of heterogeneity between the contestants. Specifically, when heterogeneity is high, appointing a jury favoring a less talented contestant and, hence, levelling the playing field will maximize the aggregate effort. At the same time, if the designer aims at maximizing the probability of a more talented contestant winning, she does equally well with any possible jury composition. When heterogeneity is low, a jury favoring a more talented contestant promotes the highest probability of his winning, although the designer does not necessarily strive to infinitely increase the bias size expecting favoritism towards a more talented contestant. Meanwhile, the aggregate effort is the same for each possible jury composition.

Keywords: contest design, committee composition, optimal bias

JEL Codes: C72, D44

1 Introduction & Related literature

It is common practice to delegate prize allocation in rent-seeking contests to a committee of experts. For instance, a firm's owner frequently devolves a CEO hiring decision to a Board of directors. At university, an admission committee

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is assigned to accept appropriate candidates for a PhD program. In some professional sports, e.g., dancesport and figure skating, athletes are rewarded for their performance by a panel of judges.

A reasonable explanation why such delegation occurs is because of a potential bias of a decision maker towards particular contestants due to either her intrinsic tastes and attitudes, in the absence of direct performance measurements, or payoffs that follow after a favorable contestant winning. When first considered, the appointment of a committee aims at elimination of biases of individual experts. Meanwhile, not every aggregation technique provides unbiased rewarding, and fair prize allocation is not necessarily an end in itself of such delegation. On the contrary, the control of aggregate bias of the committee members proves to be an instrument for manipulating the efforts of competitors.

In this paper, we ask what jury composition is optimal to appoint by the contest designer if jury members are potentially biased. To answer the question of interest, we propose a stylized theoretical model. Our study contributes to the theory unifying optimal biased contests and decision-making in committees by introducing biased jury members. Perhaps surprisingly, the unbiased committee composition is suboptimal to stimulate the contestants. Meanwhile, only a small bias might be sufficient to maximize chances of the stronger contestant winning.

The proposed theoretical model is based on a sequential move game. By backward induction, we first define the equilibrium efforts of two heterogeneous contestants competing for a single prize in the lottery contest. The chance to win may be increased for one of the contestants by a bias provided by an appointed jury of potentially biased reviewers via voting procedure. Next, we study the equilibrium strategy of each jury member in the voting game. The contest designer moves first and decides on the jury composition and the optimal bias size.

We found that the clue of the optimal contest is degree of heterogeneity between the contestants. If the degree of heterogeneity is sufficiently large, the contest designer appoints a jury that favors a weaker contestant to create even playing field if she benefits from the total effort. If the degree of heterogeneity is sufficiently small, the contest designer appoints a jury that favors a stronger contestant to strengthen his chances of winning if that is her objective point.

This paper is related to two research fields. First, it stands close to the literature on the optimal biased contests. Second, our study is strongly associated with the literature on decision making in committees. Accordingly, the main contribution of the study is to the literature that unifies contests and decision-making committees, in particular, considering optimal bias provided by members of a jury appointed by the contest designer.

A substantial body of theoretical literature on the rent-seeking concept, i.e., contests, originates from Tullok (1980). To date, a great deal of previous research has focused on various design-related questions, e.g., optimal contest format, optimal prize-allocation across stages in dynamic tournaments, optimal seeding rule in elimination contests, and many others. Regarding the question addressed, it seems worth noticing the existing results on biased contests and contests with head starts. The effect of head starts on multiprize contest model are examined in (Siegel 2014). With identical valuations and homogeneous prizes proposed, the equilibrium strategic profile is characterized by weaker contestants' more aggressive behavior than of stronger ones.

Favoritism is studied in an asymmetric contest framework in (Kirkegaard 2012). It has been shown that contestants' heterogeneity determines an instrument that proves to be optimal to regulate the asymmetry. Providing a head start to a weaker contestant allows to increase total effort exerted by the contestants, but in some cases, the maximum total effort can be obtained by introducing both handicap and head start to a weaker contestant. The optimality of employing handicap to a stronger contestant is also not intuitively straightforward and depends on the degree of heterogeneity among contestants. In (Fu and Wu 2020) the optimal design of biased contests is studied concerning various instruments, such as implementing additive head start and multiplicative bias. Efficacy analysis of different favoritism instruments is investigated in (Franke et al. 2018).

In all these studies, implementing an instrument to manipulate efforts and the contest outcome is deterministic, i.e., the contestants are aware of the head start or bias provision. We study a contest setting where the employment of the bias is private information of the jury.

For an extensive review of the theoretical results on decision-making in committees, see (H. Li and Suen 2009). The review covers, among other things, the optimal committee composition problem treating committee members as heterogeneous in preferences and possessing private information of their preferences. In (Yildirim et al. 2018), the study focuses on the optimal composition of a committee of biased experts from the perspective of a status quo biased principal. Optimal composition proves to be non-monotonic in the majority rule. In (H. Li, Rosen, et al. 2001), information sharing is studied when committee members have conflicting preferences. Information manipulation and strategical voting are examined as motivated by reputational regards in committees with experts having different preferences in (Visser and Swank 2007). Decision-making of careerist experts is studied from the focus of transparency in (Levy 2007a; Levy 2007b).

A search of the literature revealed few studies which put decision-making committee within the Tulluck contest framework. The first discussion of such a contest format is in (Congleton 1984). The study compares equilibrium strategies between contests under committee administration and with a single administrator. In (Amegashie 2002), Congleton's model is modified in voting by committee members is treated as probabilistic. The evidence obtained in (Amegashie 2003), where the author develops the model by introducing caps on the efforts, shows that in some cases (with properly chosen caps) aggregate effort can be higher than in the case of a single administrator. In (Lockard 2006), Congleton's results are also reexamined considering a proportionate-sharing rule. Committee composition issue is raised in (Amegashie 2006), but the focus is on comparing known and unknown composition. Optimal committee size in such a contest format is investigated in (S. Li et al. 2013).

So, while some research has been carried out on committee decision making within a rent-seeking contest framework, there is need to study how introducing biased reviewers as jury members affects the contest result, when members of a jury decide on the bias provision. We showed that the clue to the optimal contest is degree of heterogeneity between the contestants.

The paper proceeds as follows. Section 2 describes the theoretical model and the equilibrium behavior of the contestants and the jury members. Section 3 represents the optimal contest considering both the optimal bias size and the optimal jury composition for each utility specification of the designer. Section 4 concludes.

2 Model Setup

There are two contestants, indexed by $i \in \{1,\ 2\}$, competing for an indivisible prize by choosing effort $e_i \geq 0$, simultaneously and independently. Let the cost function of contestant i be linearly increasing, and the contestants are heterogeneous with respect to the marginal costs, i.e., $c_i(e_i) \coloneqq c_i e_i$. Suppose $0 < c_1 < c_2$, so the first contestant may be viewed as a more talented one. Each contestant is risk neutral, values the prize at 1 and the loss at 0, and chooses the effort to maximize expected payoff:

$$\pi_i(e_i) \coloneqq P_i - c_i e_i,$$

where $P_i \coloneqq P_i(e_1,\ e_2,\ b)$ is the probability contestant i wins, defined as:

$$P_i(e_1,\ e_2,\ b) \coloneqq \begin{cases} \frac{e_i}{e_i+e_{-i}}, & \text{if bias is not provided, } e_1^2+e_2^2>0; \\ \frac{be_i}{be_i+e_{-i}}, & \text{if bias is provided to contestant } i,\ e_1^2+e_2^2>0; \\ \frac{e_i}{e_i+be_{-i}}, & \text{if bias is provided to contestant } -i,\ e_1^2+e_2^2>0; \\ \frac{1}{2}, & \text{if } e_1^2+e_2^2=0. \end{cases}$$

We use a modification of the Tullock success function that allows for a multiplicative[1] bias $b > 1$ (see Franke et al. 2018, Fu and Wu 2020). Either it is provided to one of the contestants and increases his probability of winning or it is not provided to anyone at all. If both contestants exert zero effort, the prize goes to each of them with equal probability.

The bias provision may be blocked by the contest designer or it can be delegated to a jury of experts, appointed by the designer. Suppose such a jury consists of three reviewers, indexed by $j \in \mathcal{J} := \{1,\ 2,\ 3\}$. For $j \neq 3$, two types are possible: biased towards contestant $i = j$ or unbiased. Jury member $j = 3$ is assumed unbiased. Denote biased type by j_b and unbiased type by $j_{\bar{b}}$. A biased reviewer benefits from her favorite's winning. Assuming risk-neutrality, let the utility of the biased type coincide with the probability of a corresponding contestant winning. A jury member does not benefit from the bias provision if she turns out to be unbiased, but is better off if the bias is not provided. So, the utility of jury member j is given by:

$$v_j(e_1, e_2, b) := \begin{cases} P_j(e_1, e_2, b), & \text{if jury member } j = j_b; \\ 0, & \text{if jury member } j = j_{\bar{b}} \text{ and the bias is provided;} \\ 1, & \text{if jury member } j = j_{\bar{b}} \text{ and the bias is not provided.} \end{cases}$$

Denote the probability of the first (resp. second) jury member being biased with p (resp. q):

$$Pr(1 = 1_b) = p \in \{0,\ 1\},$$
$$Pr(2 = 2_b) = q \in \{0,\ 1\}.$$

We also assume that the types of the two potentially biased jury members are uncorrelated.

It is up to the contest designer to determine size of the bias b as well as the jury composition. We consider two standard specifications of a utility function the designer may be willing to maximize. Either the designer profits from the total effort exerted by the contestants:

$$u_1(b) := e_1(b) + e_2(b),$$

or the designer profits from winning of a more talented contestant:

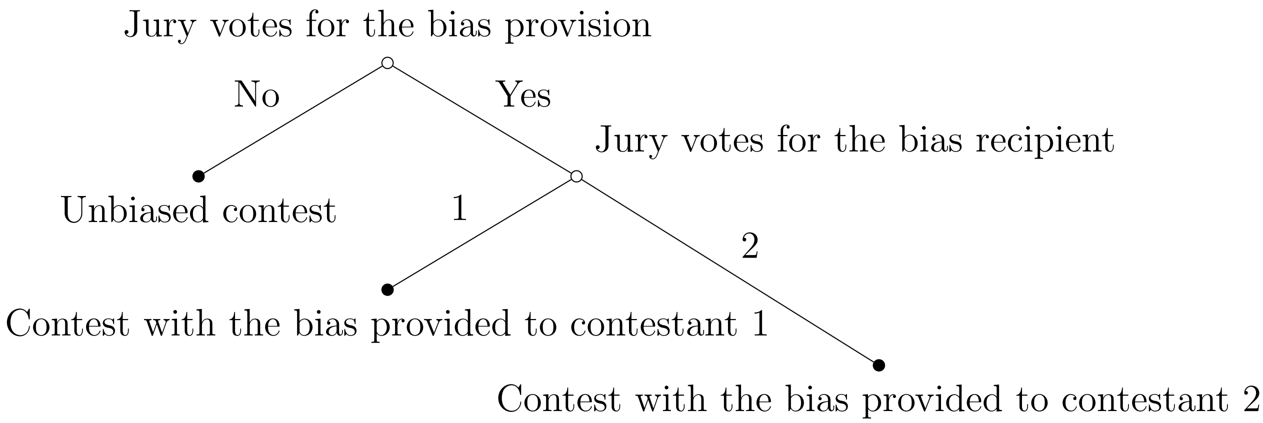
$$u_2(b) := P_1(b).$$

To maximize utility, the designer can choose among four jury compositions: (i) $p = q = 1$, (ii) $p = 1,\ q = 0$, (iii) $p = 0,\ q = 1$, and (iv) $p = q = 0$.

[1] We do not consider provision of an additive head start $a_i \in \mathbb{R}$ to manipulate the contest outcome, as any designer-optimal linear transformation $f_i(e_i) = a_i + b_i e_i$ can be replaced with $\tilde{f}_i(e_i) = \tilde{b}_i e_i$, i.e., $\tilde{a}_i = 0$ and $\tilde{f}_i(e_i) = f(e_i)$ (see Theorem 2 in Fu and Wu 2020).

If the bias provision is allowed by the designer, i.e., it is optimal to set $b^* > 1$, the bias provision is delegated to the appointed jury. Firstly, the jury decides whether any of the contestants receives the bias via majority vote. If the provision is accepted, they next vote for the bias recipient[2]. Assume strategic voting. Figure 1 demonstrates the scheme of sequential voting.

Figure 1: Scheme of the Voting Process



The contestants exert efforts being unaware of the jury's decision, but they observe the appointed jury composition.

The model presents a sequential move game. The game proceeds as follows:

1. the designer appoints a jury and decides to either allow the bias provision and choose optimal size $b > 1$ or block the bias provision;

2. if the bias provision is not blocked, the jury sequentially votes for the bias provision and the bias recipient;

3. the contestants choose optimal effort to exert.

We use backward induction to solve for equilibrium behavior of the players.

2.1 Equilibrium in the game between contestants

Since the contestants are not aware of the bias provision, we introduce $\theta_i \in [0, 1]$ to denote the probability contestant i is a bias recipient. The complementary $1 - \theta_1 - \theta_2 \in [0, 1]$ indicates the probability the bias is not provided. Then for each pair $\theta_1,\ \theta_2$, contestant i solves:

$$\max_{e_i \geq 0} \left\{\theta_i \frac{be_i}{be_i + e_{-i}} + \theta_{-i} \frac{e_i}{e_i + be_{-i}} + (1 - \theta_1 - \theta_2) \frac{e_i}{e_i + e_{-i}} - c_i e_i\right\}.$$

[2] The jury votes sequentially to break the Condorcet cycle.

The following proposition describes the equilibrium behavior of the contestants.

Proposition 1. *For any θ_1, θ_2 and $b > 1$, the Nash equilibrium of the contest game is unique. In this equilibrium, contestant i exerts the effort of $e_i^*(b,\ \theta_1,\ \theta_2)$:*

$$e_i^*(b,\ \theta_1,\ \theta_2) = \frac{1}{c_i} \cdot \frac{c_1}{c_2} \left[\frac{\theta_1 b}{(b + \frac{c_1}{c_2})^2} + \frac{\theta_2 b}{(1 + b\frac{c_1}{c_2})^2} + \frac{1 - \theta_1 - \theta_2}{(1 + \frac{c_1}{c_2})^2} \right],$$

and the equilibrium probabilities of winning are:

$$P_1^*(b,\ \theta_1,\ \theta_2) = \theta_1 \frac{b}{b + \frac{c_1}{c_2}} + \theta_2 \frac{1}{1 + b\frac{c_1}{c_2}} + (1 - \theta_1 - \theta_2)\frac{1}{1 + \frac{c_1}{c_2}},$$

$$P_2^*(b,\ \theta_1,\ \theta_2) = \theta_1 \frac{\frac{c_1}{c_2}}{b + \frac{c_1}{c_2}} + \theta_2 \frac{b\frac{c_1}{c_2}}{1 + b\frac{c_1}{c_2}} + (1 - \theta_1 - \theta_2)\frac{\frac{c_1}{c_2}}{1 + \frac{c_1}{c_2}}.$$

Proof. See Appendix. □

The result is obtained by solving a system of first order conditions, which sufficiency is guaranteed by strict convexity of the objective functions. For both contestants, an outside option, i.e., to exert zero effort, is a strictly dominated strategy.

Let us briefly describe the comparative statics of the contestants' equilibrium effort. In equilibrium, the costs incurred are equal, i.e., $c_1 e_1^*(b,\ \theta_1,\ \theta_2) = c_2 e_2^*(b,\ \theta_1,\ \theta_2)$. The ratio $\frac{c_1}{c_2} \in (0, 1)$ reflects a degree of heterogeneity between the contestants. The lower the ratio of marginal costs, which implies the higher degree of heterogeneity, the closer the equilibrium efforts become to 0, and the equilibrium probabilities of winning are balanced in favor of contestant 1. If the ratio $\frac{c_1}{c_2}$ is close to 1, which corresponds to the case of homogeneity, the contestants exert approximately equal efforts and the equilibrium probabilities of winning depend only on the probabilities of the bias provision θ_1 and θ_2. The closer the bias size to 1, the less θ_1 and θ_2 affect an outcome of the game.

Following the concept of sequential equilibrium proposed by Kreps and Wilson (1982), we now treat θ_1 and θ_2 as beliefs the contestants hold about the bias provision. We are able to solve for the fully consistent beliefs immediately after equilibria in the game between the jury members are defined.

2.2 Equilibrium in the sequential voting game of the jury members

Suppose the bias provision is not blocked by the contest designer, and the jury decides on it.

Let σ_{tj} denote a vote cast by jury member j in voting $t \in \{1,2\}$. In the first voting, the jury votes for provision of the bias, no matter who is a recipient. Let $\sigma_{1j} \in \{Y, N\}$, where Y stands for vote 'Yes', i.e., provide the bias, and N stands for 'No'.

Let $\sigma_{2j} \in \{1,2\}$ indicate who of the contestants is proposed by jury member j as the bias recipient. A profile of votes in voting t is denoted by σ_t, $\sigma_1 \in \{Y, N\}^3, \sigma_2 \in \{1,2\}^3$. Let $\sigma = (\sigma_1, \sigma_2)$ be a notion for a profile of votes in the two sequantial votings. Let σ be equivalently an element of a set $\Sigma := \{\{Y, N\} \times \{1,2\}\}^3$.

Our second result is a set of equilibria in a sequential voting game of the jury members for each possible composition given by p and q. Only pure strategies are considered.

Proposition 2. *For any $p + q > 0$, there exist an equilibrium profile $\sigma^* \in \Sigma$ supporting a system of beliefs $\theta_1^* \in [0,1],\ \theta_2^* \in [0,1] :\ \theta_1^* + \theta_2^* = 1$.*

- *Each equilibrium profile of votes σ^*, such that $\exists\ j_1 \neq j_2$: $\sigma^*_{1j_1} = \sigma^*_{1j_2} = Y$ and $\sigma^*_{2j_1} = \sigma^*_{2j_2} = 1$ implies $\theta_1^* = 1,\ \theta_2^* = 0$ as the fully consistent beliefs.*
- *For equilibrium profile of votes σ^*, such that $\exists\ j_1 \neq j_2$: $\sigma^*_{1j_1} = \sigma^*_{1j_2} = Y$ and $\sigma^*_{2j_1} = \sigma^*_{2j_2} = 2$ the fully consistent beliefs are $\theta_1^* = 0,\ \theta_2^* = 1$.*
- *Each equilibrium profile of votes σ^*, such that $\exists\ j_1 \neq j_2$: $\sigma^*_{1j_1} = \sigma^*_{1j_2} = N$ implies $\theta_1^* = \theta_2^* = 0$ as the fully consistent beliefs.*

Proof. See Appendix. □

So, if the designer delegates the bias provision to a jury with at least one biased reviewer, we can expect an equilibrium with the bias provided to one of the contestants (see the list of equilibria for each p, q in Appendix). In every equilibrium obtained, no jury member is better off by deviating from her votes cast. The bias provision is a possible equilibrium outcome of the voting game only under unanimity of the jury members.

It is also worth noticing that the presence of a single biased jury member, say, favoring contestant i, does not necessarily imply contestant $-i$ never receiving the bias in equilibrium, though two times less likely to occur if take the equilibria as equiprobable. This may happen because of indifference of the remaining unbiased jury members when at first they cast votes for the bias provision. Since they are indifferent, they can both cast vote in favor of contestant $-i$, so the biased jury member cannot affect the bias provision in favor of contestant i.

Consistent beliefs of the contestants about an outcome of the voting game are defined by implementing the Bayes' rule. The fact that the beliefs θ_1^* and θ_2^* for each equilibrium profile of votes σ are fully consistent results from the existence of a sequence $\varepsilon_k \to 0$, such that a sequence of the jury members' trembled strategies $\sigma(\varepsilon_k) \to \sigma$ induces the sequences of the probabilities defined be the Bayes' rule $\theta_1(\varepsilon_k) \to \theta_1^*$ and $\theta_2(\varepsilon_k) \to \theta_2^*$.

Substituting the fully consistent beliefs $\theta_1^*,\ \theta_2^*$ into the expression for the equilibrium effort $e_i(b,\ \theta_1^*,\ \theta_2^*)$, defined in Proposition 1, along with a corresponding equilibrium profile of votes σ, gives a sequential equilibrium. If it is believed that the bias is provided to contestant 1, the equilibrium effort of contestant i is given by:

$$e_i^*(b) = \frac{1}{c_i} \cdot \frac{c_1}{c_2} \frac{b}{(b + \frac{c_1}{c_2})^2}.$$

If the contestants believe that the bias is provided to contestant 2, the equilibrium effort of contestant i is defined as:

$$e_i^*(b) = \frac{1}{c_i} \cdot \frac{c_1}{c_2} \frac{b}{(1 + b\frac{c_1}{c_2})^2}.$$

Belief that the bias is not provided results in the following equilibrium effort of contestant i:

$$e_i^*(b) = \frac{1}{c_i} \cdot \frac{c_1}{c_2} \frac{1}{(1 + \frac{c_1}{c_2})^2}.$$

The obtained equilibrium behaviors of the contestants and the jury members allow us to determine the optimal design of such a contest.

3 The Optimal Contest

The designer is able to manipulate the contest outcome by blocking the bias provision or delegating it to the jury of experts with a fixed $b > 1$. Appointment a certain composition of the jury, given by p and q, is another tool the designer has to gain maximum utility. Note that blocking the bias provision is equivalent to setting $b = 1$ or appointing the jury of unbiased experts, i.e., $p = q = 0$.

3.1 The designer maximizes the total effort

First, we determine optimal size of the bias b^* for each p, q if the designer benefits from the total effort exerted by the contestants. To resolve uncertainty about an equilibrium realizing in the voting game, we assume each one occurs with equal probability and the designer is risk-neutral. So, the following problem must be solved:

$$\max_{b \geq 1} \Big\{ w_1 u_1(b,\ 1,\ 0) + w_2 u_1(b,\ 0,\ 1) + (1 - w_1 - w_2) u_1(b,\ 0,\ 0) \Big\},$$

where w_i stands for the probability contestant i receives the bias. Here, $b^* = 1$ essentially means blocking the bias provision, and $b^* > 1$ corresponds to a setting where the bias provision is delegated to the jury of experts.

If count the equilibria for $p = q = 1$, $w_1 = w_2 = 2/12$. Both contestants receive the bias with the same probability.

Proposition 3. *Consider a jury with two biased experts, i.e., $p = q = 1$, and the designer, seeking to maximize the total effort. For $\frac{c_1}{c_2} \geq 2 - \sqrt{3}$, the designer blocks the bias provision, that is $b^* = 1$. For $\frac{c_1}{c_2} < 2 - \sqrt{3}$, the designer delegates the bias provision to the jury, and the optimal bias is:*

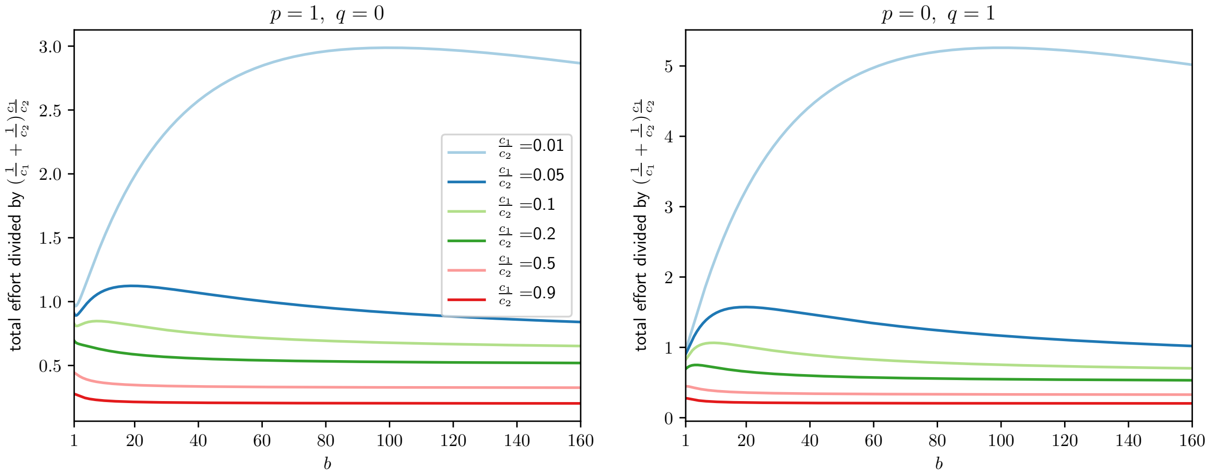
$$b^* = \frac{\frac{\left(\frac{c_1}{c_2}\right)^4 - 6\left(\frac{c_1}{c_2}\right)^2 + 1}{\frac{c_1}{c_2}\left[1 + \left(\frac{c_1}{c_2}\right)^2\right]} + \sqrt{\left(\frac{\left(\frac{c_1}{c_2}\right)^4 - 6\left(\frac{c_1}{c_2}\right)^2 + 1}{\frac{c_1}{c_2}\left[1 + \left(\frac{c_1}{c_2}\right)^2\right]}\right)^2 - 4}}{2} > 1.$$

Proof. See Appendix. □

Proposition 3 indicates that the bias provision may both positively and negatively affect the contestants' incentives under uncertainty about the voting game result. If the degree of heterogeneity is sufficiently small, i.e., $\frac{c_1}{c_2}$ is close to 1, non-zero chances to receive the bias discourage the contestants to exert higher efforts. Hence, the contest designer benefits from status quo blocking the jury providing the bias. Meanwhile, the sufficiently large degree of heterogeneity, i.e., $\frac{c_1}{c_2}$ is close to 0, implies the contest designer setting the bias $b^* > 1$ to provoke the contestants play more aggressively. Moreover, the greater the degree of heterogeneity, the higher is the optimal bias.

Unfortunately, the analytical solution for $p = 1,\ q = 0$ and $p = 0,\ q = 1$ is not as easy to derive as for $p = q = 1$. Instead, we found a numerical solution to these jury compositions. Figure 2 demonstrates the same effect of the bias provision.

Figure 2: The Total Effort Exerted by the Contestants as a Function of the Bias Size and The Jury Composition



Note: this figure indicates existence of the bias $b^* > 1$ maximizing the designer's utility for sufficiently large degree of heterogeneity between the contestants. The left plot is for the jury with one member biased towards the first contestant, the right plot is for the jury with one member biased towards the second contestant. In both simulations, b takes values between 1 and 160 with a step of 0.1.

Result 1. *If degree of heterogeneity between the contestants is sufficiently large, there is $b^* > 1$ that maximizes the total effort exerted by the contestants. For the sufficiently small degree of heterogeneity, the contest designer blocks the bias provision.*

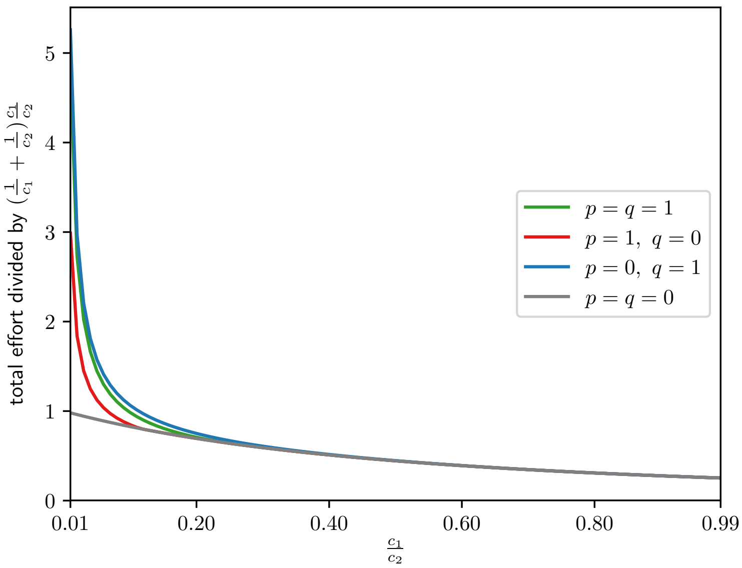
To simulate behavior of the utility function of the designer, we substituted $w_1,\ w_2$. Note that $w_i = 2w_{-i}$ if the jury composition is in favor of contestant i. The total effort is calculated excluding a multiplier $(\frac{1}{c_1} + \frac{1}{c_2})\frac{c_1}{c_2} > 0$ to avoid setting exact values for $c_1,\ c_2$ as their ratio is what really affects the contest outcome.

One can notice that for $\frac{c_1}{c_2} = 0.2$ the jury composition with a single expert favoring contestant 1 implies blocking the bias provision, while another composition still requires the rational designer to set $b^* > 1$. This may serve as an evidence the critical level of heterogeneity is greater for $p = 0,\ q = 1$ than for $p = 1,\ q = 0$.

Another important observation is that the maximum value of the designer's utility is noticeably greater for the case $p = 0,\ q = 1$, but for the small degree of heterogeneity this is not evident. Figure 2 also shows that the optimal bias size exists, i.e., it is finite. So, by using the bias the designer creates an even playing field, to equalize the ex ante heterogeneous contestants, encourage them to exert greater efforts and consequently obtain greater utility.

To complete our analysis of the optimal contest for the case where the total effort is maximized, we determine the optimal jury composition if b is fixed at its optimal level b^* for each composition. Figure 3 represents numerical simulations for this specification of the designer's utility.

Figure 3: Maximum Total Effort for Different Jury Compositions and the Degree of Heterogeneity Between the Contestants



Note: this figure shows order of the designer's maximum utility for feasible compositions of the jury. Maximum total effort is divided by $(\frac{1}{c_1}+\frac{1}{c_2})\frac{c_1}{c_2} > 0$ (exclusion of this multiplier obviously does not distort the ordinal relations). For compositions $p \neq q$ it is calculated as the maximum value of a function on the set of b taking values from a range from 1 to 160 with a step of 0.1. The ratio $\frac{c_1}{c_2}$ takes values between 0.01 and 0.99 with a step of 0.01.

Result 2. *If the contest designer benefits from the total effort exerted by the contestants, it is optimal to appoint the jury composed of two unbiased experts and one favoring a less talented contestant. Moreover,*

$$u_1(p = 0,\ q = 1) \geq u_1(p = q = 1) \geq u_1(p = 1,\ q = 0) \geq u_1(p = q = 0).$$

If the degree of heterogeneity between the contestants is sufficiently large, the non-strict inequalities become strict; if the degree of heterogeneity is sufficiently small, the designer becomes indifferent to the composition of the jury.

The large degree of heterogeneity between the contestants implies that the designer has to level the playing field, if she maximizes the total effort, by delegating the bias provision to the jury of biased experts. The less is the role of the jury member favoring a more talented contestant, the greater is the total effort. However, the presence of a biased jury member, whoever of the contestants is her favorite, is strictly better than a jury of all unbiased experts. If the degree of heterogeneity is sufficiently small, the designer has no incentive to allow the bias provision.

3.2 The designer maximizes the probability of a more talented contestant winning

Next, we study how the optimal bias needs to look like if the designer maximizes the probability that a more talented contestant wins. Similar to the previous case, we assume the equilibria in the voting game are equiprobable.

Proposition 4. *Consider the designer seeking to maximize the probability of contestant 1 winning. She blocks the bias provision whenever* $q = 1$*. For* $p = 1,\ q = 0$*, and* $\frac{c_1}{c_2} < \frac{1}{\sqrt{2}}$*, the designer delegates the bias provision to the jury, and the optimal bias is:*

$$b^* = \frac{-\frac{c_1}{c_2} + \sqrt{2}\left[\left(\frac{c_1}{c_2}\right)^2 - 1\right]}{2\left(\frac{c_1}{c_2}\right)^2 - 1} > 1.$$

For $p = 1,\ q = 0$*, and* $\frac{c_1}{c_2} \geq \frac{1}{\sqrt{2}}$*, no solution exists, i.e.,* $b^* \to \infty$*.*

Proof. See Appendix. □

Presence of a jury member favoring a less talented contestant makes the bias provision suboptimal. Still the degree of heterogeneity affects the optimal bias size. If the degree of heterogeneity is sufficiently large, the rational designer sets $b^* > 1$, and any $b > b^*$ decreases the probability of contestant 1 winning. Meanwhile, if the contestants are sufficiently close in terms of their marginal costs, the rational designer strives to set as high b as possible.

Our final result is the optimal jury composition if b is fixed at its optimal level b^* and the designer maximizes the probability of a more talented contestant winning. Figure 4 represents numerical simulations for this specification of the designer's utility.

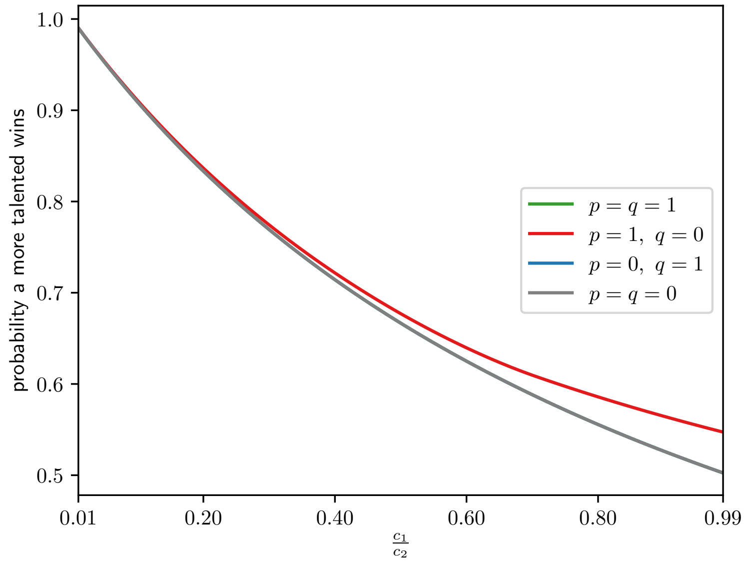
Result 3. *If the contest designer strives to maximize the probability of a more talented contestant winning, it is optimal to appoint the jury composed of two unbiased experts and one favoring a more talented contestant. Moreover,*

$$u_2(p = 1,\ q = 0) \geq u_2(p = q = 1) = u_2(p = 1,\ q = 0) = u_2(p = q = 0).$$

If the degree of heterogeneity is sufficiently small, the non-strict inequality becomes strict; if the degree of heterogeneity is sufficiently large, the designer becomes indifferent to the composition of the jury.

So, the degree of heterogeneity has an opposite effect if the designer maximizes the probability of contestant 1 winning. The sufficiently small degree

Figure 4: Maximum Probability of Contestant 1 Winning for Different Jury Compositions and the Degree of Heterogeneity



Note: this figure shows order of the designer's maximum utility for feasible compositions of the jury. The ratio $\frac{c_1}{c_2}$ takes values between 0.01 and 0.99 with a step of 0.01.

implies the optimal appointment of a jury with a single biased expert who favors a more talented contestant. Any other composition of the jury is suboptimal. We can expect any jury composition if the degree of heterogeneity is sufficiently large.

4 Conclusion

This paper aimed at determining optimal composition of a jury, appointed by the contest designer, in presence of biases towards contestants the jury members may hold. We found that, if degree of heterogeneity between the contestants is sufficiently large, the rational designer, striving to maximize the total effort, appoints a jury with an expert biased towards a less talented contestant to encourage the contestants by creating level playing field. However, if the degree of heterogeneity is sufficiently small, the designer is indifferent. The opposite effect was revealed if the designer maximizes the probability of a more talented contestant winning. Sufficiently small degree of heterogeneity implies that it is optimal to allow a jury with an expert biased towards a more talented contestant to affect the contest outcome. Meanwhile, sufficiently large degree of heterogeneity makes the designer indifferent.

Further research may be carried out by implementing several refinements of Nash equilibrium in the voting game. For now, we restricted our focus on the pure strategies of the jury members, and we expect to obtain a more stable and therefore more plausible equilibrium outcome by considering the mixed strategies. The theoretical results of the study may be tested on data from dancesport competitions. Then they can serve as a reference point to analyze contests with biased jury members from optimal design perspective.

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Appendix

Proof of ***Proposition 1****.* First, consider the case $e_1^2 + e_2^2 \neq 0$.

Expected payoffs of the contestants are given by:

$$\pi_1 = \theta_1 \frac{be_1}{be_1 + e_2} + \theta_2 \frac{e_1}{e_1 + be_2} + (1 - \theta_1 - \theta_2)\frac{e_1}{e_1 + e_2} - c_1 e_1,$$

$$\pi_2 = \theta_1 \frac{e_2}{be_1 + e_2} + \theta_2 \frac{be_2}{e_1 + be_2} + (1 - \theta_1 - \theta_2)\frac{e_2}{e_1 + e_2} - c_2 e_2.$$

Contestant i maximizes the expected payoff by choosing $e_i > 0$. Taking derivative w.r.t. e_i for each $i \in \{1,\ 2\}$ gives us the following system of FOCs:

$$\frac{\partial \pi_1}{\partial e_1} = \theta_1 \frac{be_2}{(be_1 + e_2)^2} + \theta_2 \frac{be_2}{(e_1 + be_2)^2} + (1 - \theta_1 - \theta_2)\frac{e_2}{(e_1 + e_2)^2} - c_1 = 0 \Leftrightarrow$$
$$\Leftrightarrow e_2 \left(\frac{\theta_1 b}{(be_1 + e_2)^2} + \frac{\theta_2 b}{(e_1 + be_2)^2} + \frac{1 - \theta_1 - \theta_2}{(e_1 + e_2)^2} \right) = c_1$$

$$\frac{\partial \pi_2}{\partial e_2} = \theta_1 \frac{be_1}{(be_1 + e_2)^2} + \theta_2 \frac{be_1}{(e_1 + be_2)^2} + (1 - \theta_1 - \theta_2)\frac{e_1}{(e_1 + e_2)^2} - c_2 = 0 \Leftrightarrow$$
$$\Leftrightarrow e_1 \left(\frac{\theta_1 b}{(be_1 + e_2)^2} + \frac{\theta_2 b}{(e_1 + be_2)^2} + \frac{1 - \theta_1 - \theta_2}{(e_1 + e_2)^2} \right) = c_2$$

So, in the equilibrium, $e_2^* = \frac{c_1}{c_2} e_1^*$. First solve w.r.t. e_1^*, then substitute into the equality:

$$e_1^* = \frac{1}{c_1} \cdot \frac{c_1}{c_2} \left[\frac{\theta_1 b}{(b + \frac{c_1}{c_2})^2} + \frac{\theta_2 b}{(1 + b\frac{c_1}{c_2})^2} + \frac{1 - \theta_1 - \theta_2}{(1 + \frac{c_1}{c_2})^2} \right],$$

$$e_2^* = \frac{1}{c_2} \cdot \frac{c_1}{c_2} \left[\frac{\theta_1 b}{(b + \frac{c_1}{c_2})^2} + \frac{\theta_2 b}{(1 + b\frac{c_1}{c_2})^2} + \frac{1 - \theta_1 - \theta_2}{(1 + \frac{c_1}{c_2})^2} \right].$$

The equilibrium expected payoffs are:

$$\pi_1^* = \theta_1 \frac{b^2}{\left(b + \frac{c_1}{c_2}\right)^2} + \theta_2 \frac{1}{\left(1 + b\frac{c_1}{c_2}\right)^2} + (1 - \theta_1 - \theta_2)\frac{1}{\left(1 + \frac{c_1}{c_2}\right)^2} > 0,$$

$$\pi_2^* = \theta_1 \frac{\left(\frac{c_1}{c_2}\right)^2}{\left(b + \frac{c_1}{c_2}\right)^2} + \theta_2 \frac{\left(b\frac{c_1}{c_2}\right)^2}{\left(1 + b\frac{c_1}{c_2}\right)^2} + (1 - \theta_1 - \theta_2)\frac{\left(\frac{c_1}{c_2}\right)^2}{\left(1 + \frac{c_1}{c_2}\right)^2} > 0.$$

Now, we show that the pair $e_1^* > 0$ and $e_2^* > 0$ is indeed a unique Nash equilibrium. If contestant i exerts $e_i > 0$ and contestant $-i$ exerts zero effort, the payoffs are $1 - c_i e_i$ and 0 respectively. If $e_1 = e_2 = 0$, the payoffs are $\frac{1}{2}, \frac{1}{2}$. It is easy to see that $e_i = 0$ is a strictly dominated strategy for each $i \in \{1, 2\}$. □

*Proof of **Proposition 2**.* In the sequential voting game between the jury members, such that

(i) $p = q = 1$, a set of Nash equilibrium votes for the bias provision is a union of the following sets:

- $\{\sigma \in \Sigma \mid \forall j \in \mathcal{J}\ \sigma_{1j} = Y,\ \sigma_{21} = \sigma_{23} = 1\}$,
- $\{\sigma \in \Sigma \mid \forall j \in \mathcal{J}\ \sigma_{1j} = Y,\ \sigma_{22} = \sigma_{23} = 2\}$,
- $\{\sigma \in \Sigma \mid \sigma_{12} = \sigma_{13} = N,\ \sigma_{21} = \sigma_{23} = 1\} \cup \{\sigma \in \Sigma \mid \sigma_{11} = \sigma_{13} = N,\ \sigma_{22} = \sigma_{23} = 2\}$;

(ii) $p = 1$, $q = 0$, a set of Nash equilibrium votes for the bias provision is a union of the following sets:

- $\{\sigma \in \Sigma \mid \forall j \in \mathcal{J}\ \sigma_{1j} = Y,\ \sigma_2 \in \{(1,1,1),(1,1,2),(1,2,1),(2,1,1)\}\}$,
- $\{\sigma \in \Sigma \mid \forall j \in \mathcal{J}\ \sigma_{1j} = Y,\ \sigma_{22} = \sigma_{23} = 2\}$,
- $\{\sigma \in \Sigma \mid \sigma_{12} = \sigma_{13} = N,\ \sigma_2 \in \{(1,1,1),(1,1,2),(1,2,1),(2,1,1)\} \cup \{\sigma \in \Sigma \mid \sigma_1 \in \{(Y,N,N),(N,N,N),(N,Y,N),(N,N,Y)\},\ \sigma_{22} = \sigma_{23} = 2\}\}$;

(iii) $p = 0$, $q = 1$, a set of Nash equilibrium votes for the bias provision is a union of the following sets:

- $\{\sigma \in \Sigma \mid \forall j \in \mathcal{J}\ \sigma_{1j} = Y,\ \sigma_{21} = \sigma_{23} = 1\}$,
- $\{\sigma \in \Sigma \mid \forall j \in \mathcal{J}\ \sigma_{1j} = Y,\ \sigma_2 \in \{(2,2,2),(2,2,1),(1,2,2),(2,1,2)\}\}$,
- $\{\sigma \in \Sigma \mid \sigma_{11} = \sigma_{13} = N,\ \sigma_2 \in \{(2,2,2),(2,2,1),(1,2,2),(2,1,2)\} \cup \{\sigma \in \Sigma \mid \sigma_1 \in \{(Y,N,N),(N,N,N),(N,Y,N),(N,N,Y)\},\ \sigma_{21} = \sigma_{23} = 1\}\}$;

(iv) $p = q = 0$, a set of Nash equilibrium votes for the bias provision is

$$\{\sigma \in \Sigma \mid \sigma_1 \in \{(N,N,N),(Y,N,N),(N,Y,N),(N,N,Y)\}\}.$$

Start with the jury $p = q = 1$. The biased contest happens if majority of the jury members voted to give the bias. If that happens, consider the equilibrium votes in the voting game 'Who is the bias recipient?'. Contestant 1 receives the bias if the jury members' votes are either of this four 111, 112, 121, 211. Contestant 2 receives the bias if the jury members votes are either of this four 222, 122, 212, 221.

In the biased contest the third jury member is indifferent, since all her payoffs are equal to 0. The equilibrium votes are 111, 121, 222, 122 (in other cases, one of the first two jury members has an incentive to deviate).

Now consider the voting game 'Is the bias provided?'. The biased contest happens if the votes are either of this four YYY, YYN, YNY, NYY. The unbiased contest occurs if the votes are either of this four NNN, YNN, NYN, NNY. The equilibrium votes are YYY, YNN, NNN for 111 or 121. The equilibrium votes are YYY, NYN, NNN for 222 or 122.

Now, we check if the obtained equilibrium profiles of votes induce fully consistent beliefs $\theta_1^*,\ \theta_2^*$.

- $(Y1, Y1, Y1, e_1^*, e_2^*), \theta_1 = 1, \theta_2 = 0$

  Introduce $\varepsilon \to 0$, $\delta \to 0$ as trembling probabilities. Completely mixed strategies for each jury member, considering their hands tremble with the same probability, are given by:

$$\sigma_{1j}^{\varepsilon} = (1-\varepsilon)Yes + \varepsilon No$$
$$\sigma_{2j}^{\delta} = (1-\delta)1 + \delta 2$$

  The consistent beliefs according to the Bayes' rule are defined as:

$$\theta_1^{\varepsilon,\delta} = (1-\varepsilon)^3(1-\delta)^3 + 3\varepsilon(1-\varepsilon)^2(1-\delta)^3 + 3(1-\varepsilon)^3\delta(1-\delta)^2 + 9\varepsilon(1-\varepsilon)^2\delta(1-\delta)^2$$

$$\theta_2^{\varepsilon,\delta} = (1-\varepsilon)^3\delta^3 + 3(1-\varepsilon)^3\delta^2(1-\delta) + 3\varepsilon(1-\varepsilon)^2\delta^3 + 9\varepsilon(1-\varepsilon)^2\delta^2(1-\delta)$$

  Take $\varepsilon_n = \delta_n = \frac{1}{n} \to 0$, then $\theta_1^{\varepsilon,\delta} \to 1, \theta_2^{\varepsilon,\delta} \to 0$. The limits coincide with the considered belief system, so the obtained is a sequential equilibrium, i.e., not only a weak sequential equilibrium.

- $(Y1, Y2, Y1, e_1^*, e_2^*), \theta_1 = 1, \theta_2 = 0$

$$\sigma_{1j}^{\varepsilon} = (1-\varepsilon)Yes + \varepsilon No$$
$$\sigma_{21}^{\delta} = \sigma_{23}^{\delta} = (1-\delta)1 + \delta 2$$
$$\sigma_{22}^{\delta} = \delta 1 + (1-\delta)2$$

  The consistent beliefs are:

$$\theta_1^{\varepsilon,\delta} = [(1-\varepsilon)^3 + 3\varepsilon(1-\varepsilon)^2][(1-\delta)^2\delta + 2\delta^2(1-\delta) + (1-\delta)^3]$$

$$\theta_2^{\varepsilon,\delta} = [(1-\varepsilon)^3 + 3\varepsilon(1-\varepsilon)^2][\delta^2(1-\delta) + 2(1-\delta)^2\delta + \delta^3]$$

Take $\varepsilon_n = \delta_n = \frac{1}{n} \to 0$, then $\theta_1^{\varepsilon,\delta} \to 1, \theta_2^{\varepsilon,\delta} \to 0$. The limits coincide with the considered belief system, so the obtained is a sequential equilibrium, i.e., not only a weak sequential equilibrium.

- $(Y2, Y2, Y2, e_1^*, e_2^*), \theta_1 = 0, \theta_2 = 1$

$$\sigma_{1j}^{\varepsilon} = (1-\varepsilon)Yes + \varepsilon No$$
$$\sigma_{2j}^{\delta} = \delta 1 + (1-\delta)2$$

The consistent beliefs are:

$$\theta_1^{\varepsilon,\delta} = [(1-\varepsilon)^3 + 3\varepsilon(1-\varepsilon)^2][\delta^3 + 3\delta^2(1-\delta)]$$
$$\theta_2^{\varepsilon,\delta} = [(1-\varepsilon)^3 + 3\varepsilon(1-\varepsilon)^2][(1-\delta)^3 + 3(1-\delta)^2\delta]$$

Take $\varepsilon_n = \delta_n = \frac{1}{n} \to 0$, then $\theta_1^{\varepsilon,\delta} \to 0, \theta_2^{\varepsilon,\delta} \to 1$. The limits coincide with the considered belief system, so the obtained is a sequential equilibrium, i.e., not only a weak sequential equilibrium.

- $(Y1, Y2, Y2, e_1^*, e_2^*), \theta_1 = 0, \theta_2 = 1$

$$\sigma_{1j}^{\varepsilon} = (1-\varepsilon)Yes + \varepsilon No$$
$$\sigma_{21}^{\delta} = (1-\delta)1 + \delta 2$$
$$\sigma_{22}^{\delta} = \sigma_{23}^{\delta} = \delta 1 + (1-\delta)2$$

The consistent beliefs are:

$$\theta_1^{\varepsilon,\delta} = [(1-\varepsilon)^3 + 3\varepsilon(1-\varepsilon)^2][(1-\delta)\delta^2 + 2(1-\delta)\delta^2 + \delta^3]$$
$$\theta_2^{\varepsilon,\delta} = [(1-\varepsilon)^3 + 3\varepsilon(1-\varepsilon)^2][\delta^2(1-\delta) + 2\delta^2(1-\delta) + (1-\delta)^3]$$

Take $\varepsilon_n = \delta_n = \frac{1}{n} \to 0$, then $\theta_1^{\varepsilon,\delta} \to 0, \theta_2^{\varepsilon,\delta} \to 1$. The limits coincide with the considered belief system, so the obtained is a sequential equilibrium, i.e., not only a weak sequential equilibrium.

- $(Y1, N1, N1, e_1^*, e_2^*), \theta_1 = 0, \theta_2 = 0$

$$\sigma_{11}^{\varepsilon} = (1-\varepsilon)Yes + \varepsilon No$$
$$\sigma_{12}^{\varepsilon} = \sigma_{13}^{\varepsilon} = \varepsilon Yes + (1-\varepsilon)No$$
$$\sigma_{2j}^{\delta} = (1-\delta)1 + \delta 2$$

The consistent beliefs are:

$$\theta_1^{\varepsilon,\delta} = [(1-\varepsilon)\varepsilon^2 + 2(1-\varepsilon)^2\varepsilon + \varepsilon^3][(1-\delta)^3 + 3\delta(1-\delta)^2]$$

$$\begin{aligned}\theta_2^{\varepsilon,\delta} &= [(1-\varepsilon)\varepsilon^2 + 2(1-\varepsilon)^2\varepsilon + \varepsilon^3][\delta^3 + 3\delta^2(1-\delta)] = \\ &= \varepsilon\delta^2[(1-\varepsilon)\varepsilon + 2(1-\varepsilon)^2 + \varepsilon^2][\delta + 3(1-\delta)]\end{aligned}$$

Take $\varepsilon_n = \delta_n = \frac{1}{n} \to 0$, then $\theta_1^{\varepsilon,\delta} \to 0, \theta_2^{\varepsilon,\delta} \to 0$. The limits coincide with the considered belief system, so the obtained is a sequential equilibrium, i.e., not only a weak sequential equilibrium.

- $(Y1, N2, N1, e_1^*, e_2^*), \theta_1 = 0, \theta_2 = 0$

$$\begin{gathered}\sigma_{11}^{\varepsilon} = (1-\varepsilon)Yes + \varepsilon No \\ \sigma_{12}^{\varepsilon} = \sigma_{13}^{\varepsilon} = \varepsilon Yes + (1-\varepsilon)No \\ \sigma_{21}^{\delta} = \sigma_{23}^{\delta} = (1-\delta)1 + \delta 2 \\ \sigma_{22}^{\delta} = \delta 1 + (1-\delta)2\end{gathered}$$

The consistent beliefs are:

$$\theta_1^{\varepsilon,\delta} = [(1-\varepsilon)\varepsilon^2 + 2(1-\varepsilon)^2\varepsilon + \varepsilon^3][(1-\delta)^2\delta + 2\delta^2(1-\delta) + (1-\delta)^3]$$

$$\theta_2^{\varepsilon,\delta} = [(1-\varepsilon)\varepsilon^2 + 2(1-\varepsilon)^2\varepsilon + \varepsilon^3][\delta^2(1-\delta) + 2(1-\delta)^2\delta + \delta^3]$$

Take $\varepsilon_n = \delta_n = \frac{1}{n} \to 0$, then $\theta_1^{\varepsilon,\delta} \to 0, \theta_2^{\varepsilon,\delta} \to 0$. The limits coincide with the considered belief system, so the obtained is a sequential equilibrium, i.e., not only a weak sequential equilibrium.

- $(N2, Y2, N2, e_1^*, e_2^*), \theta_1 = 0, \theta_2 = 0$

$$\begin{gathered}\sigma_{11}^{\varepsilon} = \sigma_{13}^{\varepsilon} = \varepsilon Yes + (1-\varepsilon)No \\ \sigma_{12}^{\varepsilon} = (1-\varepsilon)Yes + \varepsilon No \\ \sigma_{2j}^{\delta} = \delta 1 + (1-\delta)2\end{gathered}$$

The consistent beliefs are:

$$\theta_1^{\varepsilon,\delta} = [(1-\varepsilon)\varepsilon^2 + 2(1-\varepsilon)^2\varepsilon + \varepsilon^3][\delta^3 + 3(1-\delta)\delta^2]$$

$$\theta_2^{\varepsilon,\delta} = [(1-\varepsilon)\varepsilon^2 + 2(1-\varepsilon)^2\varepsilon + \varepsilon^3][(1-\delta)^3 + 3(1-\delta)^2\delta]$$

Take $\varepsilon_n = \delta_n = \frac{1}{n} \to 0$, then $\theta_1^{\varepsilon,\delta} \to 0, \theta_2^{\varepsilon,\delta} \to 0$. The limits coincide with the considered belief system, so the obtained is a sequential equilibrium, i.e., not only a weak sequential equilibrium.

- $(N1, Y2, N2, e_1^*, e_2^*), \theta_1 = 0, \theta_2 = 0$

$$\sigma_{11}^{\varepsilon} = \sigma_{13}^{\varepsilon} = \varepsilon Yes + (1-\varepsilon)No$$
$$\sigma_{12}^{\varepsilon} = (1-\varepsilon)Yes + \varepsilon No$$
$$\sigma_{21}^{\delta} = (1-\delta)1 + \delta 2$$
$$\sigma_{22}^{\delta} = \sigma_{23}^{\delta} = \delta 1 + (1-\delta)2$$

The consistent beliefs are:

$$\theta_1^{\varepsilon,\delta} = [(1-\varepsilon)\varepsilon^2 + 2(1-\varepsilon)^2\varepsilon + \varepsilon^3][(1-\delta)\delta^2 + 2(1-\delta)\delta + \delta^3]$$
$$\theta_2^{\varepsilon,\delta} = [(1-\varepsilon)\varepsilon^2 + 2(1-\varepsilon)^2\varepsilon + \varepsilon^3][(1-\delta)^2\delta + 2(1-\delta)\delta^2 + (1-\delta)^3]$$

Take $\varepsilon_n = \delta_n = \frac{1}{n} \to 0$, then $\theta_1^{\varepsilon,\delta} \to 0, \theta_2^{\varepsilon,\delta} \to 0$. The limits coincide with the considered belief system, so the obtained is a sequential equilibrium, i.e., not only a weak sequential equilibrium.

- $(N1, N1, N1, e_1^*, e_2^*), \theta_1 = 0, \theta_2 = 0$

$$\sigma_{1j}^{\varepsilon} = \varepsilon Yes + (1-\varepsilon)No$$
$$\sigma_{2j}^{\delta} = (1-\delta)1 + \delta 2$$

The consistent beliefs are:

$$\theta_1^{\varepsilon,\delta} = [\varepsilon^3 + 3\varepsilon^2(1-\varepsilon)][(1-\delta)^3 + 3(1-\delta)^2\delta]$$
$$\theta_2^{\varepsilon,\delta} = [\varepsilon^3 + 3\varepsilon^2(1-\varepsilon)][\delta^3 + 3\delta^2(1-\delta)]$$

Take $\varepsilon_n = \delta_n = \frac{1}{n} \to 0$, then $\theta_1^{\varepsilon,\delta} \to 1, \theta_2^{\varepsilon,\delta} \to 0$. The limits coincide with the considered belief system, so the obtained is a sequential equilibrium, i.e., not only a weak sequential equilibrium.

- $(N1, N2, N1, e_1^*, e_2^*), \theta_1 = 0, \theta_2 = 0$

$$\sigma_{1j}^{\varepsilon} = \varepsilon Yes + (1-\varepsilon)No$$
$$\sigma_{21}^{\delta} = \sigma_{23}^{\delta} = (1-\delta)1 + \delta 2$$
$$\sigma_{22}^{\delta} = \delta 1 + (1-\delta)2$$

The consistent beliefs are:

$$\theta_1^{\varepsilon,\delta} = [\varepsilon^3 + 3\varepsilon^2(1-\varepsilon)][(1-\delta)^2\delta + 2(1-\delta)\delta^2 + (1-\delta)^3]$$

$$\theta_2^{\varepsilon,\delta} = [\varepsilon^3 + 3\varepsilon^2(1-\varepsilon)][\delta^2(1-\delta) + 2(1-\delta)^2\delta + \delta^3]$$

Take $\varepsilon_n = \delta_n = \frac{1}{n} \to 0$, then $\theta_1^{\varepsilon,\delta} \to 1, \theta_2^{\varepsilon,\delta} \to 0$. The limits coincide with the considered belief system, so the obtained is a sequential equilibrium, i.e., not only a weak sequential equilibrium.

- $(N2, N2, N2, e_1^*, e_2^*), \theta_1 = 0, \theta_2 = 0$

$$\sigma_{1j}^{\varepsilon} = \varepsilon Yes + (1-\varepsilon)No$$
$$\sigma_{2j}^{\delta} = \delta 1 + (1-\delta)2$$

The consistent beliefs are:

$$\theta_1^{\varepsilon,\delta} = [\varepsilon^3 + 3\varepsilon^2(1-\varepsilon)][\delta^3 + 3(1-\delta)\delta^2]$$
$$\theta_2^{\varepsilon,\delta} = [\varepsilon^3 + 3\varepsilon^2(1-\varepsilon)][(1-\delta)^3 + 3\delta(1-\delta)^2]$$

Take $\varepsilon_n = \delta_n = \frac{1}{n} \to 0$, then $\theta_1^{\varepsilon,\delta} \to 1, \theta_2^{\varepsilon,\delta} \to 0$. The limits coincide with the considered belief system, so the obtained is a sequential equilibrium, i.e., not only a weak sequential equilibrium.

- $(N1, N2, N2, e_1^*, e_2^*), \theta_1 = 0, \theta_2 = 0$

$$\sigma_{1j}^{\varepsilon} = \varepsilon Yes + (1-\varepsilon)No$$
$$\sigma_{21}^{\delta} = (1-\delta)1 + \delta 2$$
$$\sigma_{22}^{\delta} = \sigma_{23}^{\delta} = \delta 1 + (1-\delta)2$$

The consistent beliefs are:

$$\theta_1^{\varepsilon,\delta} = [\varepsilon^3 + 3\varepsilon^2(1-\varepsilon)][(1-\delta)\delta^2 + 2(1-\delta)^2\delta + \delta^3]$$
$$\theta_2^{\varepsilon,\delta} = [\varepsilon^3 + 3\varepsilon^2(1-\varepsilon)][(1-\delta)^2\delta + 2\delta^2(1-\delta) + (1-\delta)^3]$$

Take $\varepsilon_n = \delta_n = \frac{1}{n} \to 0$, then $\theta_1^{\varepsilon,\delta} \to 1, \theta_2^{\varepsilon,\delta} \to 0$. The limits coincide with the considered belief system, so the obtained is a sequential equilibrium, i.e., not only a weak sequential equilibrium.

So, all the obtained equilibria are sequential equilibria.

Now, we consider the case $p = q = 0$. All jury members are unbiased, so they all are indifferent if any of the contestants receives the bias. Any combination XXX, X from {1, 2} is Nash equilibrium (NE).

The NE votes in the voting game 'Is the bias provided?' are $(\sigma_{11}, \sigma_{12}, \sigma_{13})$: NNN, YNN, NYN, NNY.

The consistent beliefs are $\theta_1 = \theta_2 = 0$.

Now, we consider the case $p = 1$, $q = 0$. The first jury member is biased towards contestant 1, the remaining jury members are unbiased.

In the voting game 'Who is the bias recipient?', the NE votes are 111, 121, 112, 211 (so, contestant 1 receives it), 122, 222 (so, contestant 2 receives it).

When they cast votes that someone receives the bias, the NE votes are YYY, YNN, NNN, if further contestant 1 is provided with the bias; YYY, NNY, NYN, YNN, NNN, if further contestant 2 is provided with the bias.

Now, we consider the case $p = 0$, $q = 1$. The second jury member is biased towards contestant 2, the remaining jury members are unbiased.

In the voting game 'Who is the bias recipient?', the NE votes are 111, 121 (so, contestant 1 receives it), 221, 212, 122, 222 (so, contestant 2 receives it).

When they cast votes that someone receives the bias, the NE votes are YYY, NNY, NYN, YNN, NNN, if further contestant 1 is provided with the bias; YYY, NYN, NNN, if further contestant 2 is provided with the bias.

Full consistency of the induced systems of beliefs supporting the derived weak sequential equilibria can be checked in the same manner as for the case $p = q = 1$. □

*Proof of **Proposition 3**.* If $p = q = 1$, there are two equilibria in the voting game, which entail the bias provision. The total effort maximization problem of the designer is equivalent to the following problem

$$\max_{b \geq 1} \; g(b) = \frac{b}{\left(b + \frac{c_1}{c_2}\right)^2} + \frac{b}{\left(1 + b\frac{c_1}{c_2}\right)^2}.$$

Firstly, we solve for local extremum points. Then we check if an obtained local maximum point is a global maximum one on the feasible set $b \geq 1$.

The first order condition is given by

$$g'(b) = \frac{\frac{c_1}{c_2} - b}{\left(b + \frac{c_1}{c_2}\right)^3} + \frac{1 - b\frac{c_1}{c_2}}{\left(1 + b\frac{c_1}{c_2}\right)^3} = 0.$$

Define $a_1 = \frac{c_1}{c_2}\left[1 + (\frac{c_1}{c_2})^2\right] \in (0, 2)$, $a_2 = (\frac{c_1}{c_2})^4 - 6(\frac{c_1}{c_2})^2 + 1 \in (-4, 1)$. Solving the FOC is equivalent to finding the roots of the following equation

$$-a_1 b^4 + a_2 b^3 - a_2 b + a_1 = 0 \;\Leftrightarrow\; (b^2 - 1)\left(b^2 - \frac{a_2}{a_1} b + 1\right) = 0$$

$$\left(\frac{a_2}{a_1}\right)^2 - 4 = 0 \Rightarrow b^2 - \frac{a_2}{a_1}b + 1 = 0 \Leftrightarrow b = \pm 1$$

$$\left(\frac{a_2}{a_1}\right)^2 - 4 > 0 \Leftrightarrow \frac{c_1}{c_2} < 2 - \sqrt{3} \Rightarrow b = \frac{\frac{a_2}{a_1} + \sqrt{\left(\frac{a_2}{a_1}\right)^2 - 4}}{2} > 1$$

is a root of interest (another root is less than 1).

The second order condition is given by

$$g''(b) = \frac{2\left(b - 2\frac{c_1}{c_2}\right)}{\left(b + \frac{c_1}{c_2}\right)^4} + \frac{2\frac{c_1}{c_2}\left(b\frac{c_1}{c_2} - 2\right)}{\left(1 + b\frac{c_1}{c_2}\right)^4} \text{ which is } \begin{cases} < 0 & b = 1,\ \frac{c_1}{c_2} > 2 - \sqrt{3}; \\ > 0 & b = 1,\ \frac{c_1}{c_2} < 2 - \sqrt{3}. \end{cases}$$

Due to the well-known behavior of a polynomial of degree 4 that takes the value 0 in four different real numbers, the obtained extremum point greater than 1, for the case $\frac{c_1}{c_2} < 2 - \sqrt{3}$, is the only global maximum point on the feasible set $b \geq 1$. So the rational designer sets the bias size at the level of

$$b^* = \frac{\frac{\left(\frac{c_1}{c_2}\right)^4 - 6\left(\frac{c_1}{c_2}\right)^2 + 1}{\frac{c_1}{c_2}\left[1 + \left(\frac{c_1}{c_2}\right)^2\right]} + \sqrt{\left(\frac{\left(\frac{c_1}{c_2}\right)^4 - 6\left(\frac{c_1}{c_2}\right)^2 + 1}{\frac{c_1}{c_2}\left[1 + \left(\frac{c_1}{c_2}\right)^2\right]}\right)^2 - 4}}{2} > 1.$$

If $\frac{c_1}{c_2} \geq 2 - \sqrt{3}$, $\forall\ b > 1\ g'(b) < 0$, and so it is optimal to block the bias provision. □

Proof of ***Proposition 4****.* If $p = q = 1$, there are two equilibria in the voting game, which entail the bias provision. The probability of a more talented contestant's winning maximization problem of the designer is equivalent to the following problem

$$\max_{b \geq 1}\ g(b) = \frac{b}{b + \frac{c_1}{c_2}} + \frac{1}{1 + b\frac{c_1}{c_2}}.$$

Firstly, we solve for local extremum points. Then we check if an obtained local maximum point is a global maximum one on the feasible set $b \geq 1$.

The first order condition is given by

$$g'(b) = \frac{c_1}{c_2}\left[\frac{1}{\left(b + \frac{c_1}{c_2}\right)^2} - \frac{1}{\left(1 + b\frac{c_1}{c_2}\right)^2}\right] = 0 \Leftrightarrow \left[\left(\frac{c_1}{c_2}\right)^2 - 1\right] b^2 = \left(\frac{c_1}{c_2}\right)^2 - 1 \Leftrightarrow b = \pm 1$$

$$g''(1) = \frac{2c_1\left(\frac{c_1}{c_2} - 1\right)}{c_2\left(1 + \frac{c_1}{c_2}\right)^3} < 0.$$

The second order condition implies $b = 1$ is a local maximum point on $\mathbb{R}$ and $g'(b) < 0$ for $b > 1$ guarantees it is the global maximum point on the set $b \geq 1$.

If $p = 1,\ q = 0$, the probability a more talented contestant receives the bias is twice greater than that a less talented one receives it, so the designer equivalently maximizes the following problem

$$\max_{b\geq 1}\ g(b) = 2\frac{b}{b+\frac{c_1}{c_2}} + \frac{1}{1+b\frac{c_1}{c_2}}.$$

The first order condition is given by

$$g'(b) = \frac{c_1}{c_2}\left[\frac{2}{\left(b+\frac{c_1}{c_2}\right)^2} - \frac{1}{\left(1+b\frac{c_1}{c_2}\right)^2}\right] = 0 \Leftrightarrow \left[2\left(\frac{c_1}{c_2}\right)^2 - 1\right]b^2 + 2\frac{c_1}{c_2}b + 2 - \left(\frac{c_1}{c_2}\right)^2 = 0$$

$$\frac{c_1}{c_2} < \frac{1}{\sqrt{2}} \Rightarrow b = \frac{-\frac{c_1}{c_2} + \sqrt{2}\left[\left(\frac{c_1}{c_2}\right)^2 - 1\right]}{2\left(\frac{c_1}{c_2}\right)^2 - 1} > 1$$

is a root of interest (another root is less than 1). Blocking the bias provision is not optimal as $g'(1) > 0$, the well-known behavior of a polynomial of degree 2 with two different real roots guarantees the obtained extremum point is the global maximum on the set $b \geq 1$. If

$$\frac{c_1}{c_2} \geq \frac{1}{\sqrt{2}} \Rightarrow \forall\ b \geq 1\ g'(b) > 0,$$

which implies there is no a finite solution, and the designer strives to set as high bias as possible.

There is no upper bound on b:

$$\lim_{b\to\infty}\pi_1(b) = \lim_{b\to\infty}\theta_1\frac{b}{b+\frac{c_1}{c_2}} + \theta_2\frac{1}{1+b\frac{c_1}{c_2}} + (1-\theta_1-\theta_2)\frac{1}{1+\frac{c_1}{c_2}} - \\ -\frac{c_1}{c_2}\left[\frac{\theta_1 b}{(b+\frac{c_1}{c_2})^2} + \frac{\theta_2 b}{(1+b\frac{c_1}{c_2})^2} + \frac{1-\theta_1-\theta_2}{(1+\frac{c_1}{c_2})^2}\right] = \theta_1 + \frac{1-\theta_1-\theta_2}{(1+\frac{c_1}{c_2})^2} \geq 0,$$

$$\lim_{b\to\infty}\pi_2(b) = \lim_{b\to\infty}\theta_1\frac{1}{b\frac{c_2}{c_1}+1} + \theta_2\frac{b}{\frac{c_2}{c_1}+b} + (1-\theta_1-\theta_2)\frac{1}{1+\frac{c_2}{c_1}} - \\ -\frac{c_1}{c_2}\left[\frac{\theta_1 b}{(b+\frac{c_1}{c_2})^2} + \frac{\theta_2 b}{(1+b\frac{c_1}{c_2})^2} + \frac{1-\theta_1-\theta_2}{(1+\frac{c_1}{c_2})^2}\right] = \theta_2 + \frac{(1-\theta_1-\theta_2)c_1}{(c_1+c_2)(1+\frac{c_2}{c_1})} \geq 0.$$

If $p = 0,\ q = 1$, the probability a less talented contestant receives the bias is twice greater than that a more talented one receives it, so the designer equivalently maximizes the following problem

$$\max_{b \geq 1}\ g(b) = \frac{b}{b + \frac{c_1}{c_2}} + \frac{2}{1 + b\frac{c_1}{c_2}}.$$

The first order condition is given by

$$g'(b) = \frac{c_1}{c_2}\left[\frac{1}{\left(b + \frac{c_1}{c_2}\right)^2} - \frac{2}{\left(1 + b\frac{c_1}{c_2}\right)^2}\right] = 0 \Leftrightarrow \left[\left(\frac{c_1}{c_2}\right)^2 - 2\right] b^2 - 2\frac{c_1}{c_2} b + 1 - 2\left(\frac{c_1}{c_2}\right)^2 = 0.$$

This quadratic equation has two roots both less than 1. Since $\forall\ b \geq 1\ g'(b) < 0$, it is optimal to block the bias provision.

If $p = q = 0$, the designer is indifferent as the jury never provides the bias.

$\square$