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**Application of the Fuzzy GARCH Model to Forecasting Volatility of the Russian Stock Market Indices**

The problem of modeling the volatility of financial instruments is considered in this research. Both scientific community and practitioners demonstrate their interest in this task. One of the possible approaches to volatility forecasting is fuzzy modeling of time series of volatility. This report is dedicated to one of the kinds of this modeling.

The family of autoregressive conditional heteroskedasticity models is the most popular approach among conventional time series models with respect to volatility modeling [1]. Models of this family often serve as benchmarks in studies aimed to the development of other tools for solving the problem. So do they in the proposed research.

The proposed in this report method can be viewed as a combination of the GARCH model and the Takagi–Sugeno fuzzy inference system [2 – 5]. Under this model, the input data space is divided into several fuzzy clusters, each of which corresponds to the “if – then” fuzzy rule. The “then” part of the rule, called consequent, has a form of the GARCH model. The “if” element of the rule, usually called antecedent, is a membership function of some kind, which define the activation extent of the corresponding cluster. The final model output is calculated as a weighted average of local models’ outputs (GARCH models which correspond to clusters), where values of the membership functions are the weights. This procedure provides the so-called soft switching between local models, which helps to avoid output “jumps” on clusters’ borders.

Fuzzy models, used for volatility forecasting, are an object of research in multiple studies. Fuzzy autoregressive conditional heteroskedasticity models are presented in such articles as [6 – 9]. In these papers authors apply models to the returns of stock indices of developing and developed economies and to the currency rates. There are also studies, dedicated to volatility modeling, which use fuzzy logic but do not rely on the GARCH paradigm: [10 – 12].

The model under consideration is applied to historical values of returns of the major stock indices of the Russian market: the MOEX Russia Index and the RTS Index. The conducted research demonstrates the competitive ability of the fuzzy GARCH model as compared to the classic generalized autoregressive conditional heteroskedasticity model. In some cases the fuzzy model shows better results, as measured by forecasting characteristics. Just as classic GARCH model modifications, the fuzzy model allows to account for such factors as clusterization and skewness of the returns series. This properties are often demonstrated by the real world financial markets. At the same time, the fuzzy GARCH model provides a greater degree of freedom for considering these phenomena, as the number of fuzzy clusters is not bounded and membership functions can be specified in different forms.

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