Optimal mechanism design of retail electricity market

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1 Introduction

It is known that the allocation of several possible price alternatives for electricity increases the social welfare obtained from the trade. This is a kind of “voluntary” price discrimination, which, as a whole, is beneficial to both society and its individual participants (for example, Ramsey prices [1, 2]). At the same time, an increase in social welfare will only occur if consumers “agree” to choose different tariff schemes, formed taking into account features of the expected load of the consumers. The goal of the present work is to offer such efficient pricing schemes for the electricity market. To this end, we use the well-known principles of the mechanism design theory. This is justified by several considerations. In interaction, there is a problem of incomplete information. Here, the electricity supplier cannot know in advance what price the consumer will prefer or what type they will allocated themselves to. The supplier has only assumptions or knows the probabilities of a particular consumer action, therefore the problem is associated with the correct “identification” of types. The mechanism created should encourage consumers to use a strategy of truthful communication of their preferences through the choice of certain prices intended specifically for them.

Recently, more and more researchers address these questions. Among other things, they are driven by the smart grid development, which enables communication with the consumer on the go. The development of online pricing schemes is becoming relevant, which is reflected in a number of works with game-theoretic formulations [3, 4, 5, 6, 7]. Currently, there are several approaches. A number of researchers determine the same price for all consumers, taking into account their strategic behavior [3, 5]. It should be noted that in this case an equilibrium similar to the Nash equilibrium will be formed. Prices are set with different levels of detail, taking into account the electrical equipment that consumers have [5, 8]. Some authors [4] propose methods of forming incentive prices with elements of optimal contracts where consumers truthfully disclose their usefulness. They also consider the formation of a single price based on the utility function, which is a combination of individual consumer preferences, and analyze the behavior strategies of individual consumers as part of an aggregated group.

In contrast to the works listed above, we propose the formation of an equilibrium that will be close to the maximum social welfare, i.e. in our case, part of the consumer surplus is not lost. It is important that the pricing mechanism we offer targets each individual consumer. Our approach combines techniques for creating a convex optimization mechanism [9]. The interaction mechanism we have created will be considered feasible if consumers voluntarily choose incentive tariffs and manage their consumption according to them.

2 Model

The consumer has information about their type θ, which belongs to the set $\theta \in \Theta$; Θ is personal information unknown to the supplier. However, there is a publicly available piece of information, which a a cumulative function of distribution of consumers by types $F(\theta)$ on the set Θ with a continuous positive density $f(\theta)$. For simplicity, assume that $\Theta \equiv [\underline{\theta}, \overline{\theta}]$.

Consumer preferences are described by the payoff function $V(\cdot)$, which depends on the supply of electricity consumed and the solutions offered on the market. In this case, for the retail electricity market *the set of solutions* $\mathcal{M}$ is a set of contracts $m(\theta) \equiv m(S(\theta), P(\theta))$ designed by the electricity supplier. The tariff for each type of θ includes the total consumption $S(\theta)$ and the price for the supplied electricity $P(\theta)$. The set of possible consumption levels is expressed by the formula $\mathcal{Q} \in \mathbb{R}_+$.

According to existing regulations, electricity is supplied and accounted for at every hour of the day $t \in T$. Besides, it has a different price at every moment t of time (hour of the day). In this regard, the utility function of the consumer depends on t but is not continuous with respect to t. T can be equal to $[1,...,24]$ by the number of hours in a day or to $[I, II, III]$ by the number of time zones during the day.

Let $m(\theta) \in \mathcal{M}$ be effective tariffs $m(\theta)$ offered by the electricity supplier. These tariffs are generated for each type of consumer. The consumer of type θ pays the sum $p(t,\theta)$ per hour t for the supply $q(t,\theta)$ of electricity.

Let $V(m(\theta),\theta)$ be the consumer payoff function. Denote by $u(t,q(t,\theta),\theta)$ the utility function of the θ-th consumer.

The consumer has the following strategies:

1. truthfully inform the supplier about their type. Then they choose the tariff $m(\theta)$ and in the period $t \in T$ get the following payoff:

$$v(t, m(\theta), \theta) \equiv u(t, q(t,\theta), \theta) - p(t,\theta), \tag{1}$$

whereas the total payoff will be

$$V(m(\theta), \theta) \equiv \sum_{t \in T} v(t, m(\theta), \theta).$$

2. pretend to be any other type $\hat{\theta}$ and choose the tariff $m\left(\hat{\theta}\right)$ obtaining in T the payoff

$$V\left(m\left(\hat{\theta}\right), \theta\right) \equiv \sum_{t \in T} \left(u\left(t, q\left(t, \hat{\theta}\right), \theta\right) - p\left(t, \hat{\theta}\right)\right). \tag{2}$$

The payoff function $V(m(\theta),\theta)$ has standard regularity properties that can be justified by generally accepted ideas about the demand for the service [**?**].

Conjecture. *The function $u(t,q(t,\theta),\theta)$ is concave downward with respect to the consumption $q(t,\theta)$, increasing and three times differentiable with respect to $q(t,\theta)$ for any pair $(t,q) \in T \times Q$.*

$$\frac{\partial u(q(t,\theta),\theta)}{\partial q(t,\theta)} \geq 0, \; \frac{\partial^2 u(q(t,\theta),\theta)}{(\partial q(t,\theta))^2} \leq 0. \tag{3}$$

This means that we have a decrease in the marginal utility of the product. The second condition will be the following:

Conjecture. *The Spence-Mirrlees condition has the form*

$$\frac{\partial u(t,q(t,\theta),\theta)}{\partial \theta} \geq 0, \; \frac{\partial u^2(t,q(t,\theta),\theta)}{\partial q(t,\theta)\, \partial \theta} \geq 0. \tag{4}$$

This condition determines the ratio of utilities of different types of consumers: with an increase in the type of consumer, the marginal utility of a unit of the consumed product is growing. The Spence-Mirrlees condition is called a single-crossing condition.

In addition, the following boundary conditions are fulfilled for the utility function:

$$u(0) = 0, \;\; u'(0) > 0, \;\; u'(\infty) \leq 0.$$

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Conjecture. *The utility function $u(t,q(t,\theta),\theta)$ is separable with respect to t.*

This condition makes it easier to solve the problem of finding the optimal mechanism when considering dependencies in individual periods.

Finally, the distribution of consumer types satisfies the following property:

Conjecture. *The characteristic*

$$w(\theta) \equiv f(\theta) / (1 - F(\theta)) \tag{5}$$

is non-decreasing.

$C(t,Q)$ is a strictly increasing convex function with respect to q:

$$C(t, Q_1) \leq C(t, Q_2), \quad Q_1 \leq Q_2; \tag{6}$$

$$C(t, \lambda \cdot Q_1 + (1-\lambda) \cdot Q_2) \leq \lambda \cdot C(t, Q_1) + (1-\lambda) \cdot C(t, Q_2), \ \forall Q_1, Q_2. \tag{7}$$

THe function $C(t, Q)$ is infinately infinitely continuously differentiable with respect to Q, $C'(t, Q) \equiv dC/dQ$ are marginal costs defined for each time period $t \in T$.

The supplier's profit is described as the difference in revenue from the supply of electricity to several consumers and the electricity purchase and transfer costs. The company maximizes the following function:

$$\pi(m(\theta)) \equiv \sum_{t \in T} \int_{\underline{\theta}}^{\overline{\theta}} p(t, \theta) \cdot f(\theta)\, d\theta - \sum_{t \in T} C\left(t, Q^t\right), \tag{8}$$

where $f(\theta)$ is the distribution function density of all consumers $F(\theta)$.

Definition. *Optimal Mechanism in dominant strategies.* Given an estimated distribution $F(\theta)$, an optimal mechanism is an optimal solution to the problem:

$$\pi(m(\theta)) \rightarrow \underset{q,p}{max}; \tag{9}$$

with the restrictions

$$\forall \theta, \hat{\theta} \in \Theta, \ \sum_{t \in T} \left(u(t, q(t, \theta), \theta) - p(t, \theta)\right) \geq 0; \tag{10}$$

$$\forall \theta, \hat{\theta} \in \Theta, \ \sum_{t \in T} \left(u(t, q(t, \theta), \theta) - p(t, \theta)\right) \geq \tag{11}$$

$$\geq \sum_{t \in T} \left(u\left(t, q\left(t, \hat{\theta}\right), \theta\right) - p\left(t, \hat{\theta}\right)\right),$$

$$\forall \theta \in \Theta, \ \forall t \in T \quad q(t, \theta) \geq 0. \tag{12}$$

Proposition 1. *(i) The optimal mechanism for pricing and determining supplies of electricity in dominant strategies (9)-(12) implements a separating equilibrium where participants truthfully disclose their types. (ii) If the solution satisfies the conditions IR and IC in each period $\forall t \in T$, then it will generally satisfy the conditions of the optimal mechanism (10)-(11).*

The solution is generated in detail for each period $\forall t \in T$. The approach to solving the problem for each period is relevant for the power industry, where there are particular characteristics of load schedules. If we compare the graph of household consumers and, for example, small industrial enterprises, we may face the following situation. In the evening, the utility of a unit of electricity to households is much greater than that of a unit of electricity to small businesses.

Further we describe the algorithm for applying the optimal mechanism, which will be implemented by each consumer to choose their contract, while the maximum social welfare is achieved.

2.1 The step-by-step algorithm of optimal pricing

Step 1. The input data is the characteristics of the electric power system, which includes the load curves of all consumers incorporated into the power system, and the characteristics of the supplier's costs. Based on the average consumption for each user, the characteristics of the utility functions (or elastic demand) in different time periods are restored.

Step 2. Determine the sequence of the consumer type levels starting with the lowest one $(\theta_1, \theta_2, ..., \theta_n)$, n - is the number of types. Solve the problem of finding the maximum social welfare. Contracts designed for $\forall \theta \in [\underline{\theta}, \overline{\theta}]$ $(S(\theta), P(\theta))$ are checked to find out if they match the corresponding types:

– for each consumer, the profitability of their contract $v(t, m(\theta), \theta)$ and someone else's $v\left(t, m\left(\hat{\theta}\right), \theta\right)$ is calculated;

– the type of consumers θ_1, for which any change of contract yields negative profitability $v\left(t,m\left(\hat{\theta}\right),\theta_1\right)<0$, $\hat{\theta}\in(\theta_2,...,\theta_n)$ is defined as the lowest;

– the next level is the type θ_2 that profits from other contracts (the contract for θ_1) more than from their own $v\left(t,m\left(\theta_1\right),\theta_2\right)\geq v\left(t,m\left(\theta_1\right),\theta_2\right)$. Other contracts turn out to be non-profitable $v\left(t,m\left(\theta_2\right),\theta_2\right)\geq v\left(t,m\left(\hat{\theta}\right),\theta_2\right)$, $\hat{\theta}\in(\theta_3,...,\theta_n)$;

– then the process continues and consumers are ranked by the profit they obtain by choosing contracts of other types.

Step 3. Based on the sorted levels of consumer types, active restrictions are determined in accordance with Proposition 2: for the lowest type, the participation restriction (10) will be active, for the rest – the consistency restrictions by types with respect to the contract of the previous consumer type (11).

Step 4. Solve the optimization problem (9)-(12) and obtain the optimal contract.

3 Conclusions

We propose an optimal mechanism based on the fulfillment of the Incentive Rationality and Incentive Compatibility conditions. We use this mechanism to set prices and, as a result, we obtain a separating equilibrium, when each consumer is inclined to choose their own prices. The solution obtained is close to the maximum welfare. This also enables optimization of the load schedule of the electric power system, which leads to more effective functioning (the scatter is reduced with respect to the daily average, pronounced peaks are smoothed out).

To formalize the model, a number of statements were proved. One of the key statements is the proposition that it is possible to use the Incentive Compatibility condition in certain periods to build a pricing mechanism for several periods. This will ensure the fulfillment of the Incentive Compatibility condition, which is most important for the optimal mechanism during the entire time interval considered. The proposed mechanism will also work in a situation when in one period of time the first consumer receives a utility from a unit of electricity higher than the second consumer, but in another period they change roles.

The mechanism was demonstrated on various configurations of a multi-consumer power system. We compared pricing schemes according to Nash, the maximum social welfare, and our mechanism.

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