

New Results in Manipulability of Social Choice Rules: Systematization and Visualization

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Extended Abstract

Manipulability is a situation when a voter or a group of voters put in the ballot insincere preferences to obtain a better social choice. It was proven almost 50 years ago by (Gibbard, 1973) and (Satterthwaite, 1975) that every not-dictatorial social choice rule is manipulable. Then, there is a question: which social choice rule is the least manipulable?

There has been many papers studying the manipulability of known social choice rules. The non-exhaustive list includes (Chamberlin, 1985), (Aleskerov, Kurbanov, 1999), (Favardin, Lepelley, 2006), (Pritchard, Wilson, 2007), (Aleskerov et. al., 2011).

The papers in this field study the degree of manipulability of social choice rules under different assumptions and with different parameters.

First, some papers study individual manipulability, and some papers study coalitional manipulability. Moreover, in the case of coalitional manipulability, there can be different assumptions regarding the ways how the coalitions may be formed.

Second, there are different probabilistic models regarding the probabilities of profiles. The two most popular are Impartial Culture (IC), when all profiles are equally likely, and Impartial Anonymous Culture (IAC), when all profiles are equally likely with respect to anonymity.

Third, there can be different number of alternatives. The cases of 3, 4 and 5 alternatives are studied in most papers.

Fourth, some papers study the case of single-valued choice, some papers study the case of multi-valued choice. The main difference comes out when there is a tie between two or more alternatives in a social choice rule. Under single-valued choice framework a tie-breaking mechanism is used, usually alphabetical tie-breaking. Under multi-valued choice ties are allowed, and each voter is assumed to have an extended preference over the set of alternatives in order to choose between any two multi-valued choices. Overall, it gives at least 5 cases (alphabetical tie-breaking with single-valued choice, 4 ways of constructing extended preferences with multi-valued choice) for the case of 3 alternatives.

Fifth, there is a wide list of manipulability indices. The most used one is the Nitzan-Kelly index (NK) which was introduced (Nitzan, 1985) and Kelly (1993). Additionally, there are indices of the freedom of manipulability ($I1^+$, $I1^-$, $I1^0$), efficiency of manipulability ($I2$ and $I3$), resoluteness (D) (Aleskerov, Kurbanov, 1999).

All these parameters lead to the complexity of the results' presentation. If one needs to find the least manipulable voting rule, he/she has to study $3 \cdot 2 \cdot 3 \cdot 5 \cdot 7 = 630$ possible models.

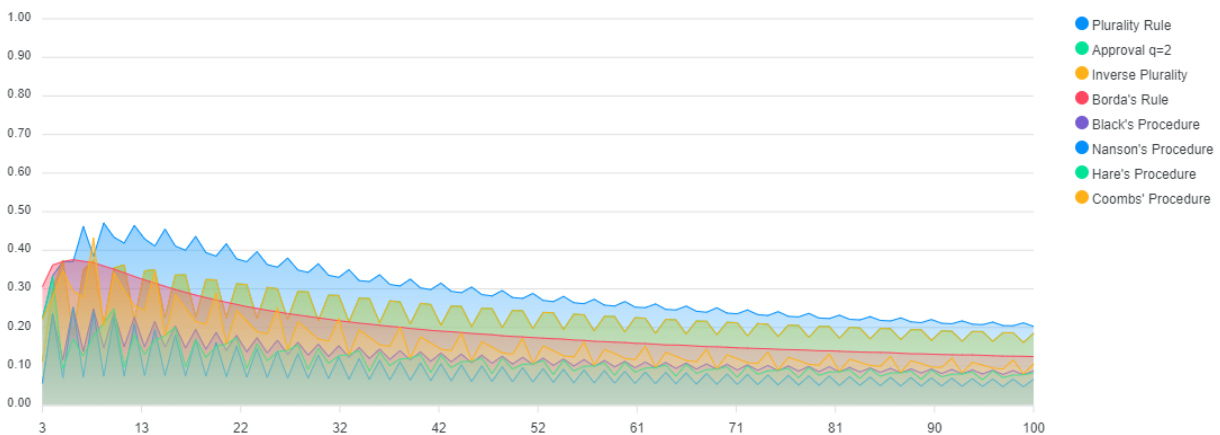
Our research consists of two parts. First, we study the manipulability of 27 known voting rules for the case of coalitional manipulability. Second, we develop a set of tools to visualize the obtained results as well as previous results in order to compare them.

In the first part we obtain the results for a new model of coalitional manipulation, which had not been studied in the literature before. We found the list of the least manipulable rules for different parameters (number of voters, extended preferences, probabilistic models).

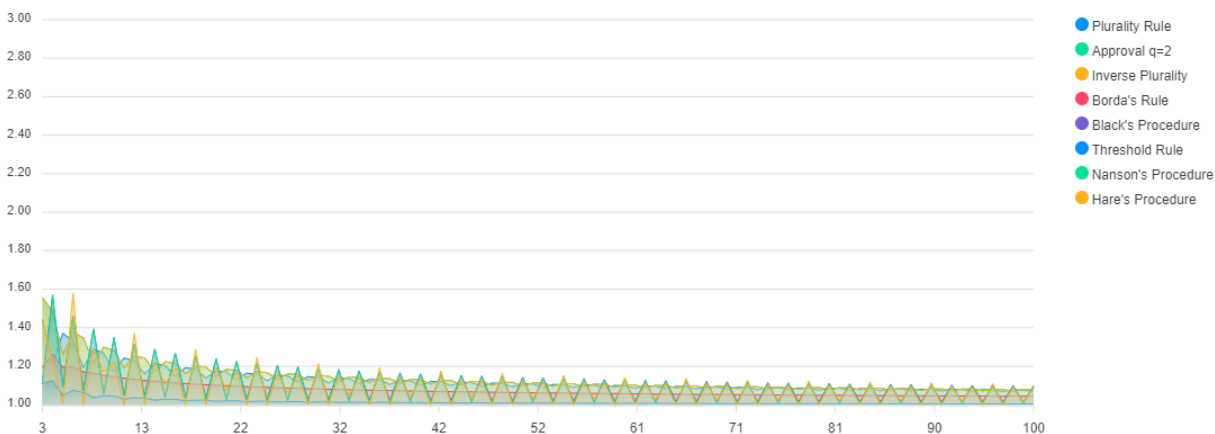
In the second part, we develop a tool allowing the visualization of the results in a convenient form. The developed tools allow analyzing not only a certain model (e.g. the model under study) with one set of parameters, but also allow the comparison of the results for different models. For example, to compare the NK indices for the case of coalitional manipulability with NK indices for the case of individual manipulability, to compare the case of alphabetical tie-breaking with the case of Leximax extended preferences, to compare indices for IC with indices for IAC,.

Let us provide some examples of how the developed tools may be useful. For example, Condorset Practical rule is known for low manipulability close to 0 if the number of voters is large. But if we take into account its resoluteness with the help of our system, it will be seen that Condorset Practical rule has low resoluteness, meaning that it gives full choice (e.g. {a, b, c} for the case of 3 alternatives), thus, does not have practical sense.

Another way of using the system is to compare rules on one or more charts. For example, we can compare the manipulability of positional voting rules for the case of coalitional manipulation (Picture 1).



Nanson's procedure and Hare's procedure show the least values of NK index, but if we compare the rules in terms of resoluteness (Picture 2)



It can be seen that the Threshold rule has the best resoluteness. Such comparisons for different cases lead to finding the combinations of the best rules in terms of manipulability and resoluteness.

More practical cases and results will be presented at the conference with live demonstration of the functionality of the system.

The next steps might be connected with integration of more results from other research groups to allow not only to analyze the data on the website, but also to upload the results for other manipulability models.

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